An anti-dispersion wave equation based on the predictor-corrector method for seismic modeling and reverse time migration

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80th SEG, 17-22 October 2010
Denver, Colorado - USA

Outline

- Motivation and introduction
- Methods
  - Wave equation - Finite difference schemes
  - Classical finite-difference (FD) and stability
  - Predictor-corrector method - 2D wave equation
  - New anti-dispersion wave equation
- Numerical examples - Modeling
- Reverse time migration results
- Conclusions
- Acknowledgments

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Tradicional wave equation

$$\frac{1}{v^2(\mathbf{x})}\frac{\partial^2 P(t,\mathbf{x})}{\partial t^2} = \nabla^2 P(t,\mathbf{x}) \qquad (1)$$

Anti-dispersion wave equation

$$\frac{1}{v^2(\mathbf{x})}\frac{\partial^2 P(t,\mathbf{x})}{\partial t^2} = \nabla^2 P(t,\mathbf{x}) + \frac{\sigma}{v(\mathbf{x})}\frac{\partial}{\partial t}\nabla^2 P(t,\mathbf{x}) \qquad (2)$$

The equation (2) was first presented by Faqi Liu et al. (2008), SEG Meeting, and later called optimized wave equation (Geophysics, vol. 74, 2009).

Introduction

- Wave equation solution based on finite-difference (FD) is a standard technique (seismic modeling and reverse time migration).
- Time step for explicit method is restrict by the stability condition. To obtain good results both spatial and temporal derivatives need to be computed with accurate operators. This can be achieved using:
  - Higher finite-difference order schemes;
  - Fine computational grids.
- On the other hand, numerical dispersion normally appears in the FD results and can contaminate the signals of interest.

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- Numerical dispersion noise is a very well known problem in FD methods - several algorithms have being proposed to obtain seismic modeling sections and migration results free from this noise.

- In this work, we propose to use the FD technique together with a predictor corrector method to obtain an efficient algorithm producing little numerical dispersion compared with the original wave equation solution.

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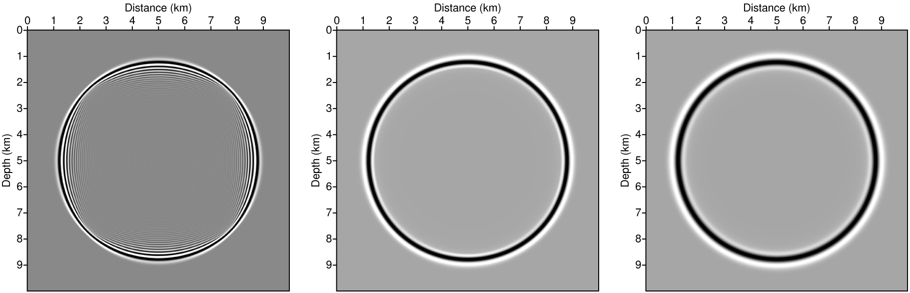
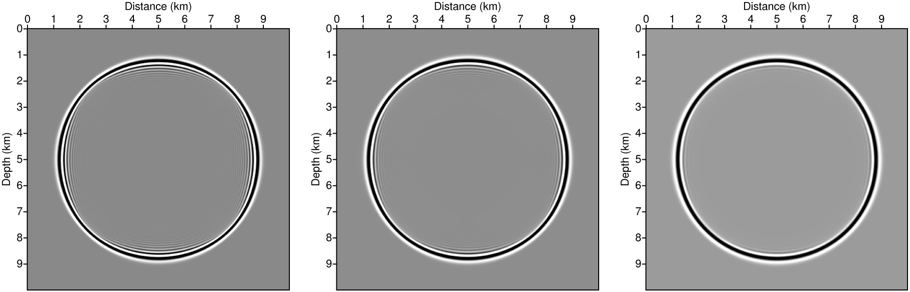
Snapshots - constant velocity model



snap1 snap2 snap3

2nd order FD scheme: Conventional method (left); anti-dispersion ($\gamma = 0.125$) (center); anti-dispersive ($\gamma = 0.5$) (left).

Snapshots - constant velocity model



snap4 snap5 snap6

2nd order FD scheme: Anti-dispersion method: $(\gamma = 0.015)$ (left); $(\gamma = 0.03)$ (center) and $(\gamma = 0.06)$ (right).

Traditional Wave Equation

For seismic modeling, we usually use the following acoustic wave equation:

$$\frac{1}{v^2(\mathbf{x})}\frac{\partial^2 P(\mathbf{x},t)}{\partial t^2} = \frac{\partial^2 P(\mathbf{x},t)}{\partial x^2} + \frac{\partial^2 P(\mathbf{x},t)}{\partial y^2} + \frac{\partial^2 P(\mathbf{x},t)}{\partial z^2} \tag{3}$$

where $P(\mathbf{x}, t)$ is the wavefield and $v(\mathbf{x})$ is the propagation velocity in the medium.

The stability condition sets a limit on the size of the time step, which for a 3-D problem, using a 2nd order scheme in time and space, gives:

$$\Delta t < \frac{min(\Delta x, \Delta y, \Delta z)}{v_{max}\sqrt{3}} \tag{4}$$

Explicit finite scheme

Using the 2nd order in time and higher-order finite differences, the forward propagation can be calculated as:

$$P_{i,j,k}^{n+1} = 2P_{i,j,k}^{n} - P_{i,j,k}^{n-1} + \Delta t^2 \, v_{i,j,k}^2 \left\{ (\nabla^2)_{i,j,k}^{M} \right\} P_{i,j,k}^{n} \qquad (5)$$

where,

$$P_{i,j,k}^{n} = P(i\Delta x, j\Delta y, k\Delta z, n\Delta t)$$

and Δt is temporal step size and Δx,Δy, Δz are spatial sampling interval.

Explicit finite scheme

The Laplacian with Mth order of accuracy can be given by

$$
\begin{aligned}
(\nabla^2)^M_{i,j,k} &= w_o\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}\right)P^n_{i,j,k} \\
&+ \sum_{m=1}^{M/2} w_m\left\{\frac{1}{\Delta x^2}\left(P^n_{i-m,j,k}+P^n_{i+m,j,k}\right)\right. \\
&+ \frac{1}{\Delta y^2}\left(P^n_{i,j-m,k}+P^n_{i,j+m,k}\right) \\
&+ \left.\frac{1}{\Delta z^2}\left(P^n_{i,j,k-m}+P^n_{i,j,k+m}\right)\right\}
\end{aligned} \tag{6}
$$

For examples, second derivative of f(x) approximated with 4th order accuracy:

$$
\begin{aligned}
f^{''}(x) &= w_2 f(x-2\Delta x) + w_1 f(x-\Delta x) + \\
&\quad w_0 f(x) + w_1 f(x-1\Delta x) + w_2 f(x-2\Delta x)
\end{aligned}
$$

Stability condition

The stability condition for isotropic modeling is as follow (Lines et al. 1999)

$$\Delta t < \frac{\Delta d}{v_{max}} \frac{2}{\sqrt{\mu}} \quad (7)$$

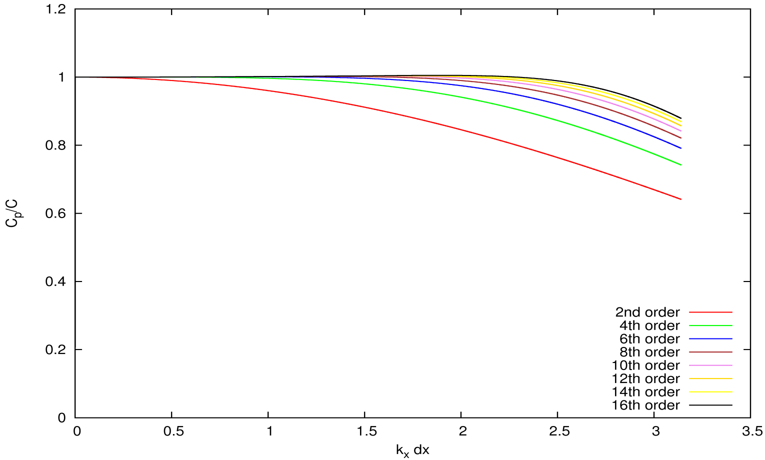
$$\mu = \sum_{m=-M/2}^{m=M/2} \left(|w_x| + |w_y| + |w_z|\right)$$

where $\Delta d = \min(\Delta x, \Delta y, \Delta z)$ and v_{max} is the maximum velocity in the medium.
The grid spacing is governed by maximum frequency or,

$$F_{max} = \frac{v_{min}}{G \Delta d}$$

and G is the number of point per shortest wavelength

Normalized phase velocity versus wavenumber for finite-difference schemes of different orders ($\alpha = 0.2$)



$C_p = \omega/k_x$ is the phase velocity and $\alpha = c\Delta t/\Delta x$.

Predictor-corrector method - Charlie's method

Let's consider the following initial value problem:

$$\frac{du}{dt} = f(u, t); \quad u(t_0) = u_0 \tag{8}$$

which can be approximated, using the explicit Euler method, by:

$$u_{n+1} = u_n + \Delta t f(u_n, t_n) \tag{9}$$

where Δt is the time step. The Charlie's algorithm to solve the equation (9) is given by Osman et al. (2007):

$$\begin{cases} \underline{Predictor}: \quad \hat{u}_n = u_n + \Delta t f(u_n, t_n) \\ \underline{Corrector}: \quad u_{n+1} = (1-\gamma)\hat{u}_n + \gamma \left[u_n + \Delta t f(\hat{u}_n, t_n) \right] \end{cases} \tag{10}$$

where $0 < \gamma < 1$ is a parameter. If $\gamma = 0$, the original explicit method is recovered by the predictor.

2D acoustic wave equation solution

The 2-D acoustic wave equation is stated according to Charlie's method as:

Predictor:

$$\hat{P} = 2P^n_{i,j} - P^{n-1}_{i,j} + r\left(P^n_{i+1,} + P^n_{i-1,j} + P^n_{i,j+1} + P^n_{i,j-1} - 4P^n_{i,j}\right) \quad (11)$$

Corrector:

$$\begin{aligned} P^{n+1}_{i,j} &= (1-\gamma)\hat{P}_{i,j} + 2\gamma\hat{P}_{i,j} + \gamma P^{n-1}_{i,j} \\ &+ \gamma\, r\left[\hat{P}_{i+1,j} + \hat{P}_{i-1,j} + \hat{P}_{i,j+1} + \hat{P}_{i,j-1} - 4\hat{P}_{i,j}\right] \quad (12) \end{aligned}$$

where $\hat{P}_{i,j}$ is a predicted value of $P^{n+1}_{i,j}$ and $r = \left(\frac{v\,\Delta t}{\Delta x}\right)^2$.

New anti-dispersion wave equation

Using a finite-difference scheme, the equation wave equation, for the 2D case, can be written as:

$$P_{i,j}^{n+1} = 2P_{i,j}^{n} - P_{i,j}^{n-1} + a\nabla^2 P_{i,j}^{n} \tag{13}$$

where $a = (v\Delta t)^2$. Using Charlie's method, the equation (13) give us the predicted value of $P_{i,j}^{n+1}$ that is:

$$\hat{P}_{i,j} = 2P_{i,j}^{n} - P_{i,j}^{n-1} + a\nabla^2 P_{i,j}^{n} \tag{14}$$

then the corrector, can be written as:

$$P_{i,j}^{n+1} = (1-\gamma)\hat{P}_{i,j} + \gamma\left[2P_{i,j}^{n} - P_{i,j}^{n-1} + a\nabla^2 \hat{P}_{i,j}^{n}\right] \tag{15}$$

New anti-dispersion wave equation

Inserting equation (14) in equation (15), and after some algebra, results in

$$P_{i,j}^{n+1} = 2P_{i,j}^{n} - P_{i,j}^{n-1} + a\nabla^2 P_{i,j}^{n} + \gamma a^2 \nabla^4 P_{i,j}^{n} + a\gamma\Delta t \frac{\partial}{\partial t}\nabla^2 P_{i,j}^{n} \quad (16)$$

Rearranging this equation and denoting $\sigma = \gamma v \Delta t$, we can recover from (16) the following partial differential equation:

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = \nabla^2 P + \frac{\sigma}{v}\frac{\partial}{\partial t}\nabla^2 P + \frac{\sigma^2}{\gamma}\nabla^4 P \quad (17)$$

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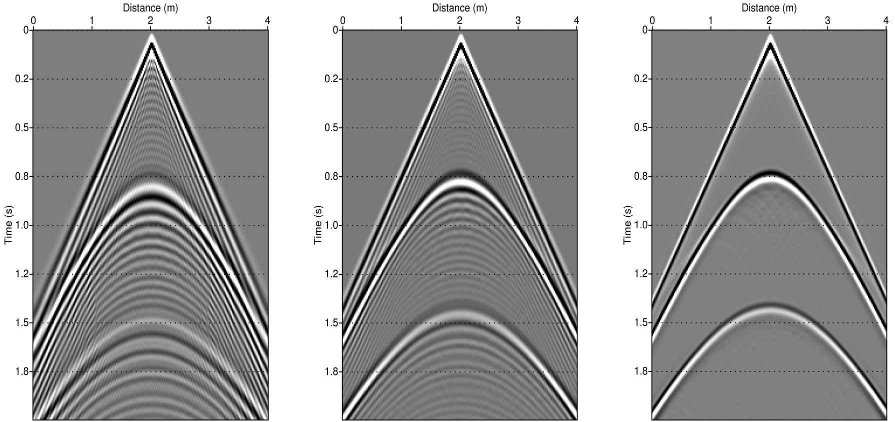
$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = \nabla^2 P + \frac{\sigma}{v}\frac{\partial}{\partial t}\nabla^2 P + \frac{\sigma^2}{\gamma}\nabla^4 P \tag{18}$$

Now, neglecting the contribution of the last term on the r.h.s. of (18), we can rewrite equation (18) as:

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = \nabla^2 P + \frac{\sigma}{v}\frac{\partial}{\partial t}\nabla^2 P \tag{19}$$

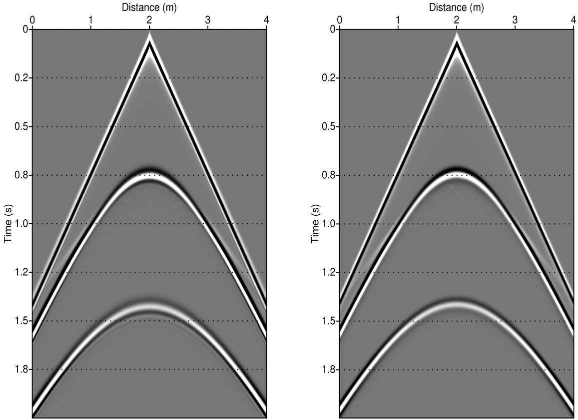
The equation (19) called anti-dispersion wave equation was first presented by Faqi Liu et al. (2009).

Syntethic seismograms -Three layers model



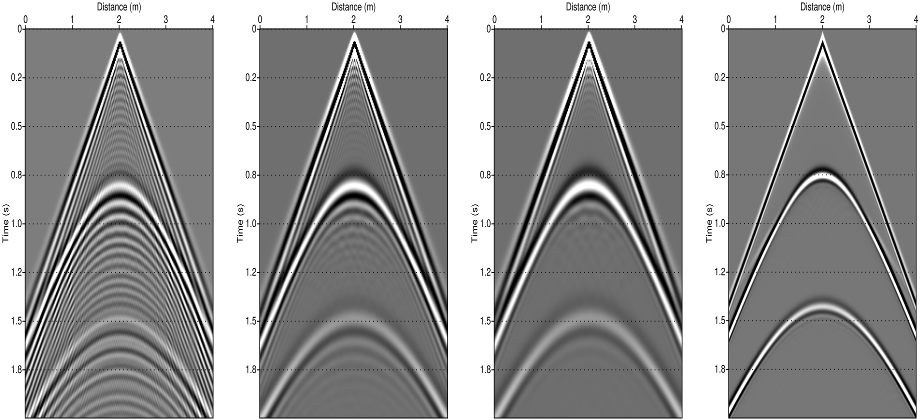
Modeled with different FD schemes with $F_{max} = 30Hz$, $\Delta x = 20m$ and 2.5 points per short wavelength. 2nd order (left), 4th order (center) and 14th order (right)

Synthetic seismograms - Three layers model



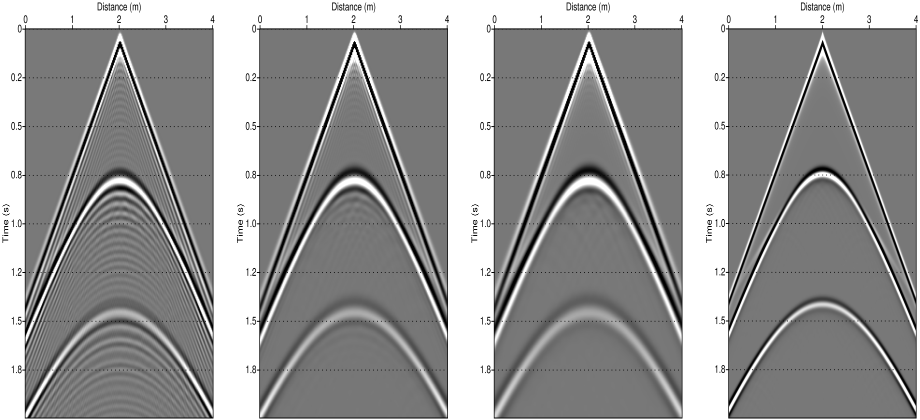
Modeled with 2nd order (left) and 4th order (right) with $F_{max} = 30Hz$ and $\Delta x = 5.0m$.

Synthetic seismograms - Three layers model

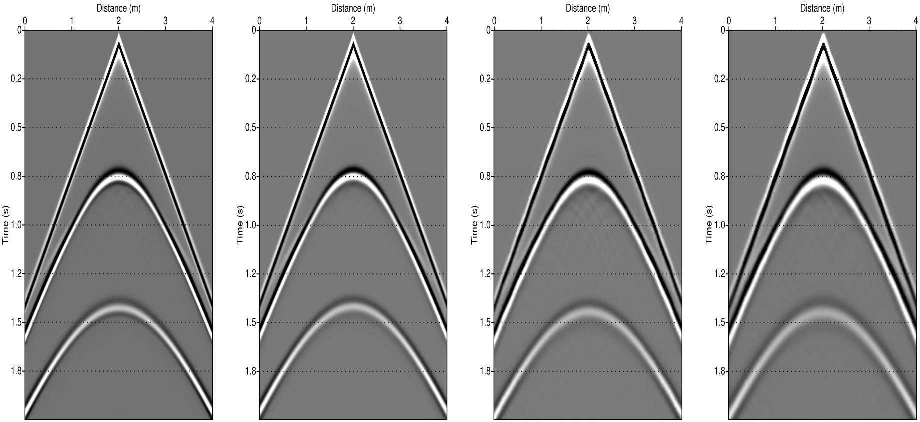


2nd order FD scheme with $F_{max} = 30Hz$ for: $\gamma = 0.0$ (1st), $\gamma = 0.125$ (2nd), $\gamma = 0.25$ (3th), $\gamma = 0.0$ and $\Delta x = 5m$ (4th).

Synthetic seismograms - Three layers model

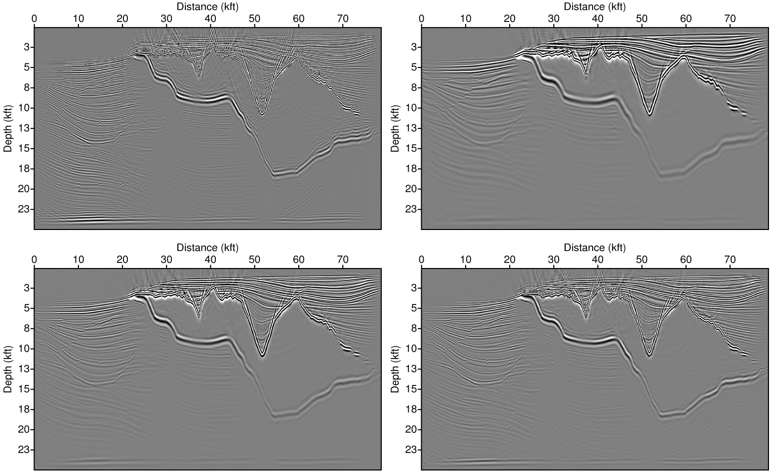


4th order FD scheme with $F_{max} = 30Hz$ for: $\gamma = 0.0$ (1st), $\gamma = 0.125$ (2nd), $\gamma = 0.25$ (3th) with $\Delta x = 20$, and $\gamma = 0.0$ and $\Delta x = 5m$ (4th)



2nd and 4th order FD schemes ($F_{max} = 30Hz$) for: $\gamma = 0.125$ (1st), $\gamma = 0.25$ (2nd) with $\Delta x = 5.0m$. 14th order FD scheme for $\gamma = 0.125$ (3rd) and $\gamma = 0.25$ (4th) with $\Delta x = 20m$.

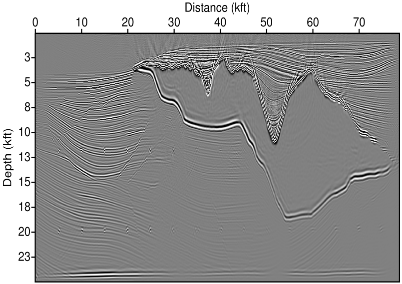
Reverse time migration results - Sigsbee2a dataset



▸ Top-left ▸ Bot-left ▸ Top-right ▸ Bot-right

4th order FD scheme with: $\gamma = 0.0$ (top left); $\gamma = 0.125$ (top right); $\gamma = 0.0625$ (bottom left); $\gamma = 0.03$ (bottom right)

Reverse time migration results - Sigsbee2a dataset



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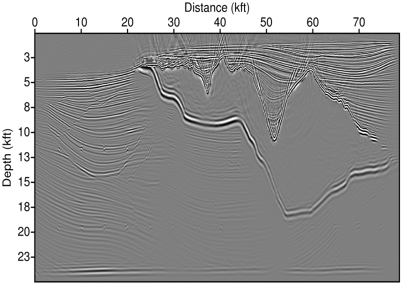

right

14th order FD scheme with: $\gamma = 0.0$ (left); and 4th order with $\gamma = 0.015$ (right).

Conclusions

- We have proposed a new acoustic wave equation based on the predictor-corrector method implemented by FD method.
- We have shown that the equation we have derived here reduces to the anti-dispersion wave equation presented by Liu et al. (2009).
- The results presented demonstrated that the new wave equation, implemented by FD schemes, can substantially attenuate the numerical dispersion compared with the original wave equation - with a small additional computational cost.
- We also tested the new method for RTM of the Sisgeeb2A with success.

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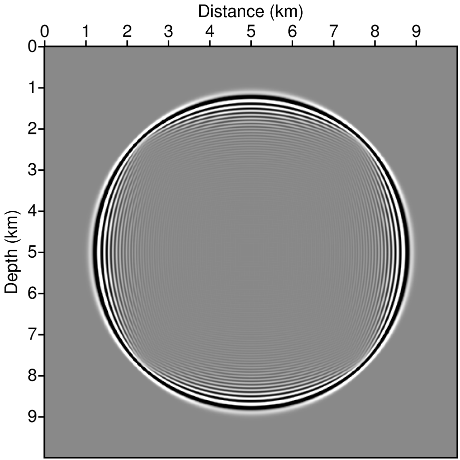
2nd order FD scheme - Conventional wave equation



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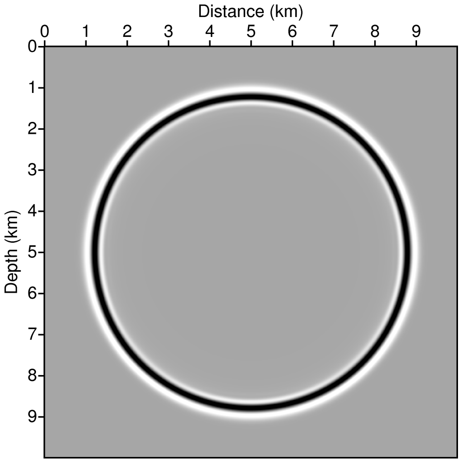
2nd order FD scheme - Anti-dispersion ($\gamma = 0.125$)



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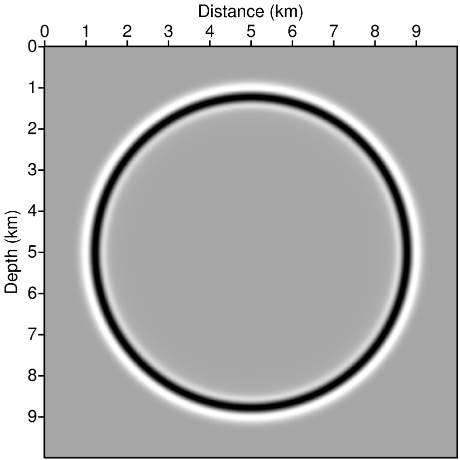
2nd order FD scheme - Anti-dispersion ($\gamma = 0.25$)



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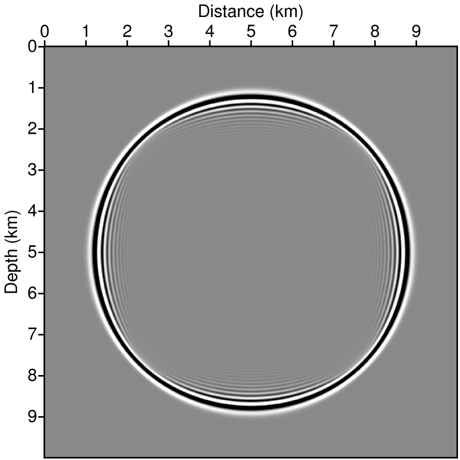
2nd order FD scheme - Anti-dispersion ($\gamma = 0.015$)



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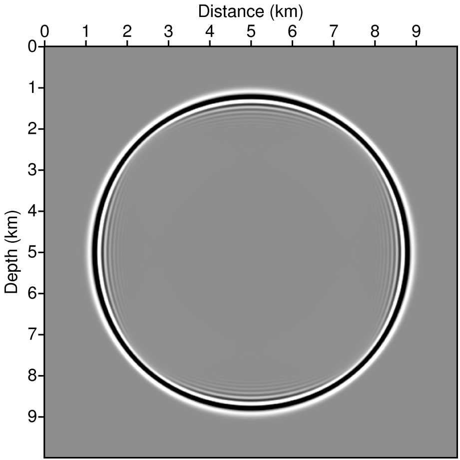
2nd order FD scheme - Anti-dispersion ($\gamma = 0.030$)



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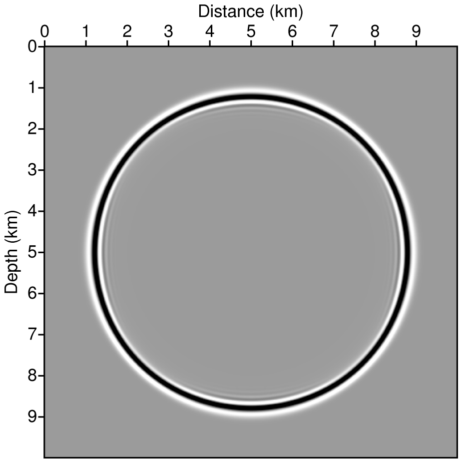
2nd order FD scheme - Anti-dispersion ($\gamma = 0.06$)



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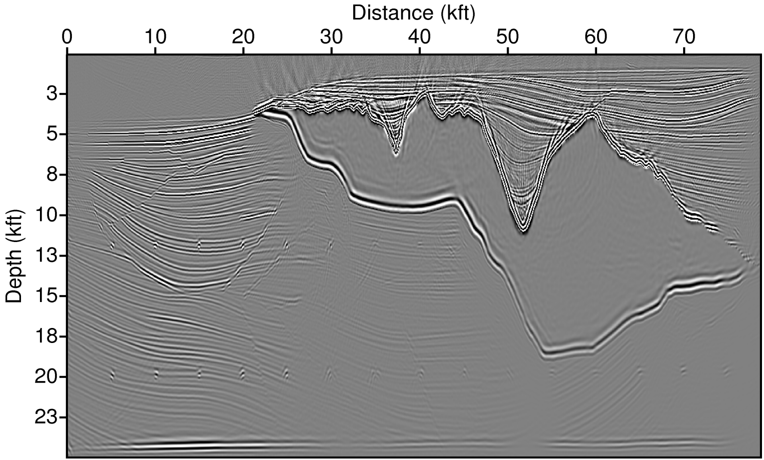
Conventional wave equation -14th order scheme



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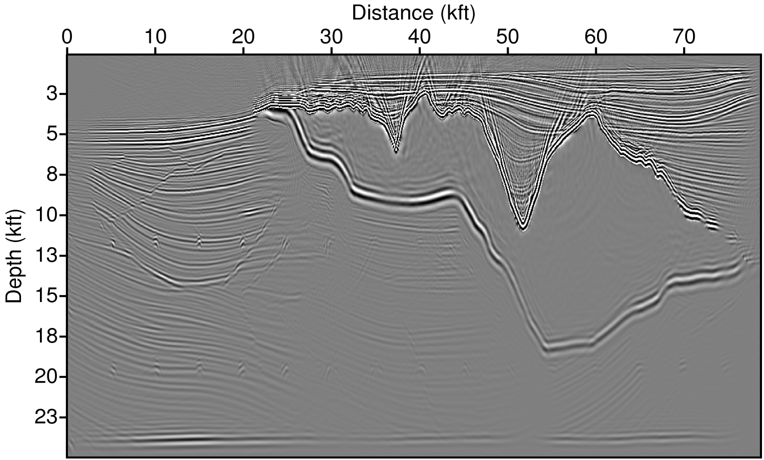
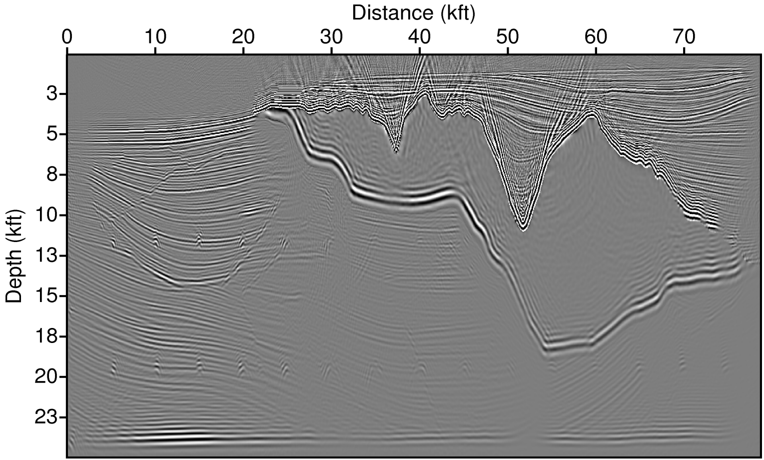
Anti-dispersion wave equation ($\gamma = 0.015$)



▸ Back ▸ 4th $\gamma = 0.0$

Conventional wave equation $\gamma = 0.0$



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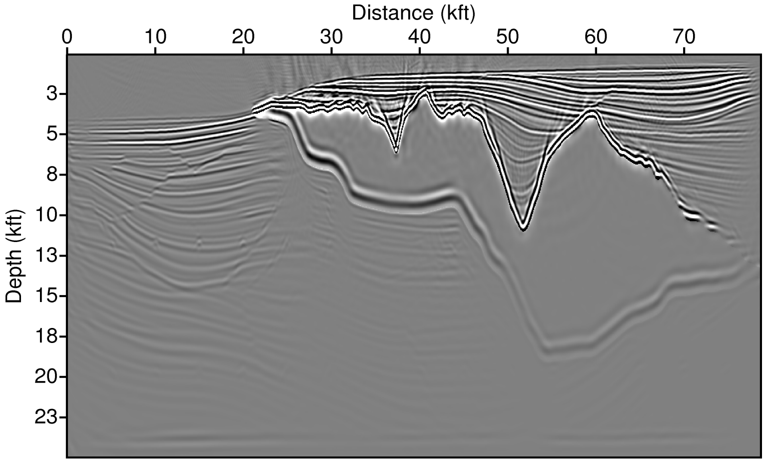
Andi-dispersion wave equation ($\gamma = 0.125$)



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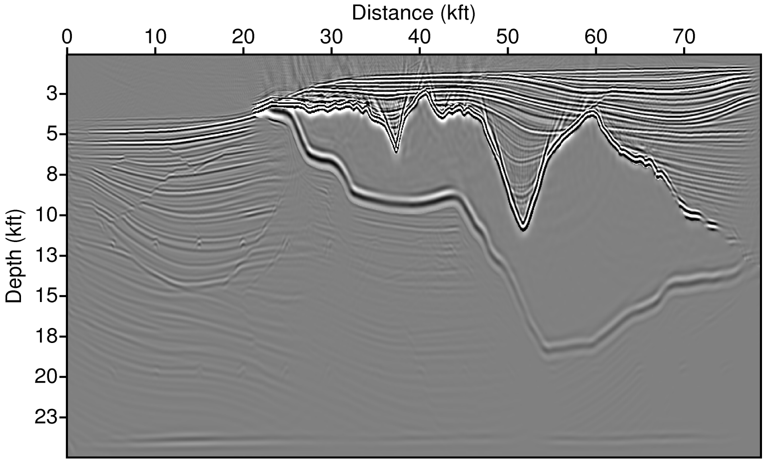
Anti-dispersion wave equation ($\gamma = 0.0625$)



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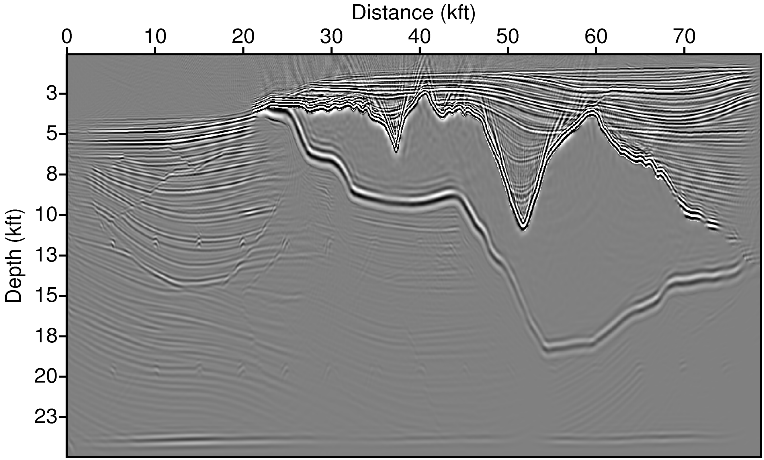
Anti-dispersion wave equation ($\gamma = 0.03$)



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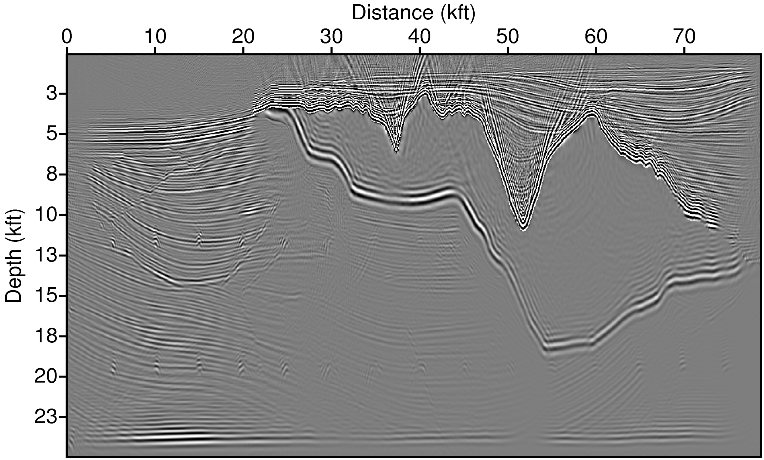
Conventional wave equation $\gamma = 0.0$



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