Reverse time migration (RTM) using analytical wavefield and causal imaging condition

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Introduction

- Reverse time migration (RTM)
  Correlation between the source (forward) and receiver (backward) wavefields.
  The image obtained is contaminated by low frequency artifacts.
- Different techniques to improve the imaging condition:
  The Laplacian filter (Guitton et al., 2007);
  Decomposition of the wavefield into one-way components (Liu et al., 2011).
- Present Method:
  (1) Apply the one-step wave-extrapolation matrix method (Pestana, 2014) for RTM problems;
  (2) Separate the analytical wavefield into its up- and downgoing components (Shen and Albertin, 2015);
  (3) Apply the causal imaging condition proposed by Liu et al. (2011).

Our method solve a first order wave-equation using the source analytical wavefield and can provide an explicit separation of the wavefield.

First order wave equation in time - Chebyshev expansion

The acoustic wave equation in two dimensional space can be written as follows

$$\frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} = \frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} \qquad (1)$$

Inserting the analytical wavefield ($\hat{P} = P(\boldsymbol{x},t) + iQ(\boldsymbol{x},t)$) into the first-order partial equation in time, we can get the following coupled set of equations (Pestana, 2014):

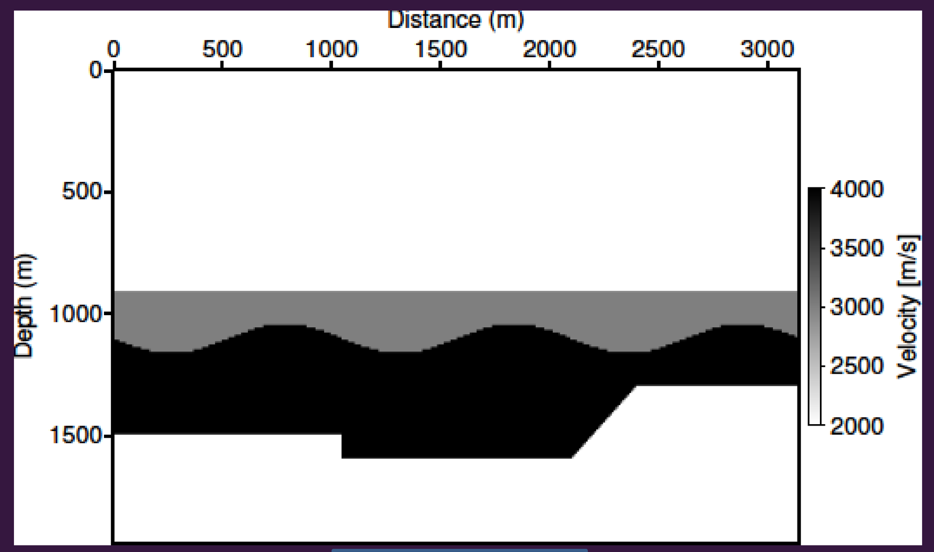
$$\frac{\partial \hat{P}}{\partial t} + i\Phi\hat{P} = 0 \qquad \Longrightarrow \qquad \frac{\partial U}{\partial t} = \boldsymbol{A}U, \qquad \text{with} \quad \boldsymbol{A} = \begin{bmatrix} 0 & \Phi \\ -\Phi & 0 \end{bmatrix} \qquad (2)$$

where $U = [P, Q]^T$ is the $2(N_x \times N_z)$ component vector of the pressure field and its Hilbert transform, $\boldsymbol{A}$ is an anti-symmetrical matrix and Φ is a pseudo-differential operator in the space domain, defined by $\phi = v(x,z)\sqrt{k_x^2 + k_z^2}$.

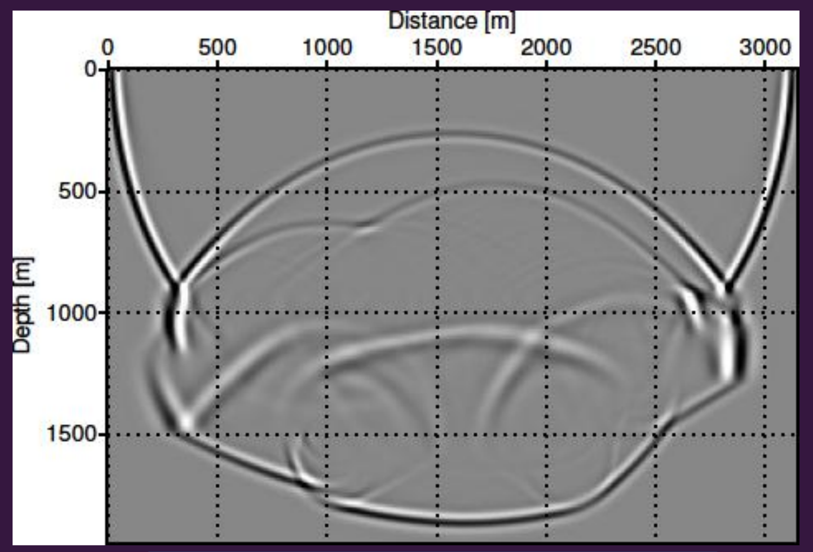
The solution of Eq. (2), including an analytical source term, can be obtained by Tal-Ezer's technique (Pestana, 2014) and it is written as:

$$\begin{bmatrix} P(t+\Delta t) \\ Q(t+\Delta t) \end{bmatrix} = \sum_{n=0}^{M} \varepsilon_n J_n(\Delta t R)\Gamma_n\left(\frac{A}{R}\right)\begin{bmatrix} P(t) \\ Q(t) \end{bmatrix} + \begin{bmatrix} f(t) \\ H\{f(t)\} \end{bmatrix} \qquad (3)$$

where J_n represents the Bessel function of order n and Γ_n are the modified Chebyshev polynomials, which satisfy the following recurrence relation: $\Gamma_{n+1}(\xi) = 2\xi\Gamma_n(\xi) + \Gamma_{n-1}(\xi)$, with $\Gamma_0(\xi) = I$ and $\Gamma_1(\xi) = \xi$.



Velocity model



Full source wavefield

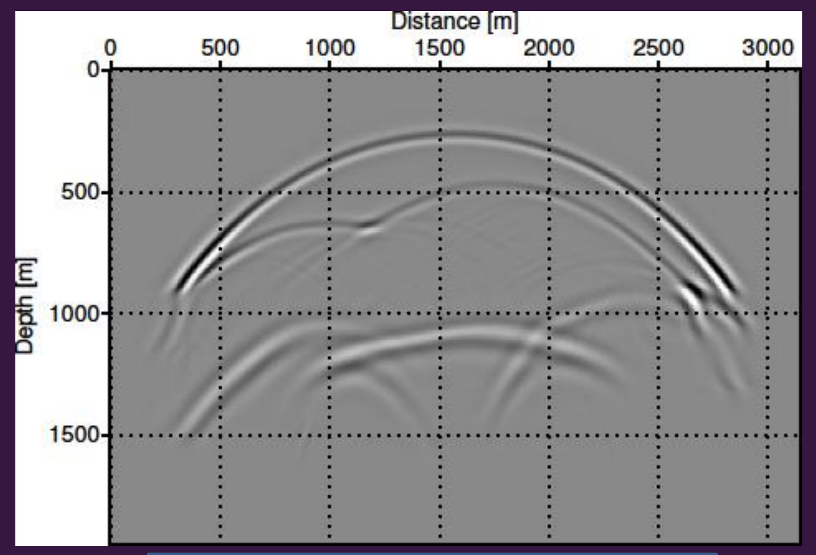
Explicit wavefield separation

The down-going component of source wavefield, S_d, in space and time becomes (Shen and Albertin, 2015)

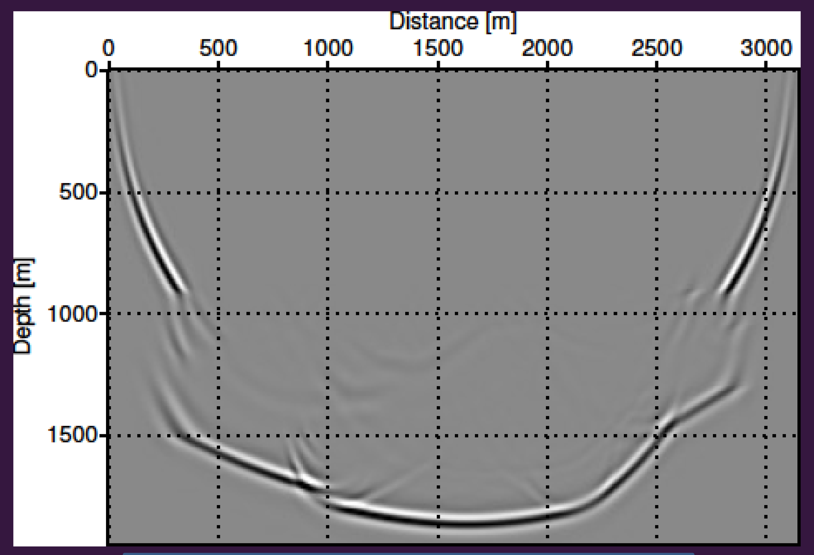
$$S_d = A = \frac{1}{2\pi} Re \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \hat{S}(x,z',t)e^{ik_z(z-z')}\kappa(k_z)dz'dk_z \qquad (4)$$

$$\kappa(k_z) = \begin{cases} 0 & if\ k_z \geq 0 \\ 1 & if\ k_z < 0 \end{cases}$$

where $\hat{S}$ is the source analytical wavefield. The up-going receiver wavefield component in forward time, R_u, is obtained using the analytical receiver wavefield in Eq. (4) with κ replaced by $1-\kappa$.



Up-going source component



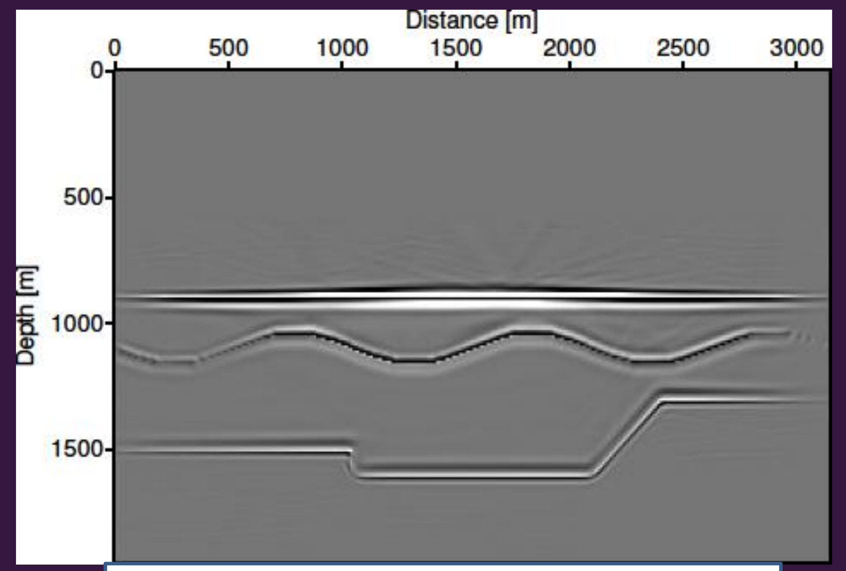
Down-going source component

Causal imaging condition

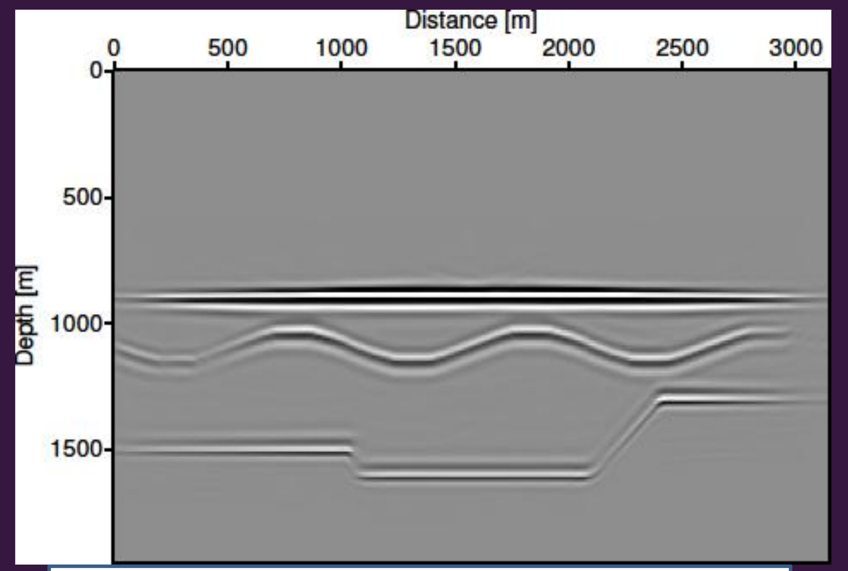
We use a causal imaging condition in which are correlated the down-going component of source wavefield, S_d, and the up-going component of the receiver wavefield, R_u (Shen and Albertin, 2015):

$$I(\boldsymbol{x}) = \int_0^T S_d(\boldsymbol{x},t)R_u(\boldsymbol{x},t)dt \qquad (5)$$

This imaging condition avoids noise along wavepaths and artifacts generated by using the conventional principle of cross-correlation.



RTM result by conventional correlation



RTM result using the causal imaging condition

Conclusions

- The one-step extrapolation method is capable to extrapolate the analytical wavefield in time up to the Nyquist time limit in stable way and free dispersion noise when the number of terms of the Chebyshev expansion is appropriately chosen.
- The causal imaging condition was implemented in the RTM algorithm. Application to synthetic data set demonstrate that it can effectively remove the undesired low-frequency noises in the image.

Acknowledgments

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