Symplectic scheme and the Poynting vector in the reverse time migration

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Main goals

- To develop a numerical solution for the acoustic wave equation and obtain the wavefield and its time derivative at the same time step.
- The proposed method applications:
  - Vector Poynting computation;
  - Wavefield separation;
  - Boundary condition problem;
  - Common imaging gathers (CIGs);
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Symplectic schemes

- In mathematics, a sympletic integrator is a numerical integration scheme for a specific group of differential equations related with classical mechanics and sympletic geometric (Yoshida, 1990).
- These schemes are widely used in molecular dynamics, celestial mechanics and other areas of physics.
- Sympletic schemes can be also used to calculate a numerical solution of the wave equation and its first time derivate.

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Wave equation - Hamiltonian system

The constant density acoustic wave equation

$$\frac{\partial^2 P}{\partial t^2} = c^2 \nabla^2 P \,, \tag{1}$$

Hamiltonian formulation of the wave equation

$$\begin{aligned} \frac{\partial P}{\partial t} &= Q \,, \\ \frac{\partial Q}{\partial t} &= c^2 \nabla^2 P \,. \end{aligned} \tag{2}$$

Leapfrog (Bonomi et. al, 1998)

$$
\begin{aligned}
Q^{(n+\frac{1}{3})} &= Q^{(n)} + \frac{1}{6}\Delta t c^2 \nabla^2 P^{(n)} \,, \\
P^{(n+\frac{1}{2})} &= P^{(n)} + \frac{1}{2}\Delta t Q^{(n+\frac{1}{3})} \,, \\
Q^{(n+\frac{2}{3})} &= Q^{(n+\frac{1}{3})} + \frac{2}{3}\Delta t c^2 \nabla^2 P^{(n+\frac{1}{2})} \,, \\
P^{(n+1)} &= P^{(n+\frac{1}{2})} + \frac{1}{2}\Delta t Q^{(n+\frac{2}{3})} \,, \\
Q^{(n+1)} &= Q^{(n+\frac{2}{3})} + \frac{1}{6}\Delta t c^2 \nabla^2 P^{(n+1)} \,.
\end{aligned}
\tag{3}
$$

Stömer-Verlet method (Chen, 2009)

$$
\begin{aligned}
Q^{(n+\frac{1}{2})} &= Q^{(n)} + \frac{1}{2}\Delta t G(P^{(n)}) \,, \\
P^{(n+1)} &= P^{(n)} + \Delta t Q^{(n+\frac{1}{2})} \,, \\
Q^{(n+1)} &= Q^{(n+\frac{1}{2})} + \frac{1}{2}\Delta t G(P^{(n+1)}) \,. \qquad (4)
\end{aligned}
$$

where

$$
G(P^{(n)}) = -c^2 F^{-1}[k^2 F(P^{(n)})] + 2\sum_{j=2}^{J} \frac{(\Delta t)^{2j-2}}{(2j)!} \frac{\partial^{2j} P^{(n)}}{\partial t^{2j}} \,. \qquad (5)
$$

Necessary and sufficient condition (Skell et al., 1997).

$$\begin{bmatrix} \frac{\partial P^{(n+1)}}{\partial P^{(n)}} & \frac{\partial P^{(n+1)}}{\partial Q^{(n)}} \\ \frac{\partial Q^{(n+1)}}{\partial P^{(n)}} & \frac{\partial Q^{(n+1)}}{\partial Q^{(n)}} \end{bmatrix}^T \begin{bmatrix} J \end{bmatrix} \begin{bmatrix} \frac{\partial P^{(n+1)}}{\partial P^{(n)}} & \frac{\partial P^{(n+1)}}{\partial Q^{(n)}} \\ \frac{\partial Q^{(n+1)}}{\partial P^{(n)}} & \frac{\partial Q^{(n+1)}}{\partial Q^{(n)}} \end{bmatrix} = \begin{bmatrix} J \end{bmatrix}. \tag{6}$$

Symplectic matrix

$$J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix}. \tag{7}$$

Analytical solution of equation (1) (Pestana and Stoffa, 2010)

$$P(t+\Delta t) + P(t-\Delta t) = 2\cos(L\Delta t)P(t)\ , \quad (L^2 = -c^2\nabla^2) \quad (8)$$

Using the REM (Kosloff et al., 1989) in (8)

$$P(t-\Delta t)+P(t+\Delta t) = 2\sum_{k=0}^{M} C_{2k} J_{2k}(\Delta t R) Q_{2k}\left(\frac{iL}{R}\right) P(t)\ , \quad (9)$$

where

$$R = c_{max}\sqrt{\left(\frac{\pi}{\Delta x}\right)^2 + \left(\frac{\pi}{\Delta y}\right)^2 + \left(\frac{\pi}{\Delta z}\right)^2} \quad (10)$$

The summation can be safely truncated with a $M > R\Delta t$ (Tal-Ezer, 1987).

Symplectic integrators and REM

Hamiltonian formulation

$$\frac{\partial P}{\partial t} = Q \qquad \text{and} \qquad \frac{\partial Q}{\partial t} = H(P)\,. \tag{11}$$

Stömer-Verlet-REM

$$\begin{aligned} P^{(n+1)} &= P^{(n)} + \Delta t Q^{(n)} + \frac{\Delta t^2}{2} H(P^{(n)})\,, \\ Q^{(n+1)} &= Q^{(n)} + \frac{\Delta t}{2}[H(P^{(n)}) + H(P^{(n+1)})]\,. \end{aligned} \tag{12}$$

where:

$$H(P^{(n)}) = \frac{2}{(\Delta t)^2}\left[\sum_{k=0}^{M} C_{2k} J_{2k}(\Delta t R) Q_{2k}\left(\frac{iL}{R}\right) - 1\right] P^{(n)}\,. \tag{13}$$

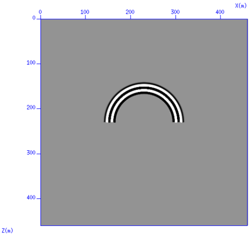
Poynting vector applications

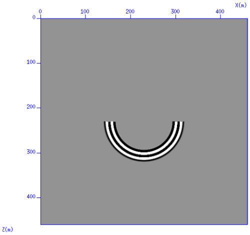
Poynting vector

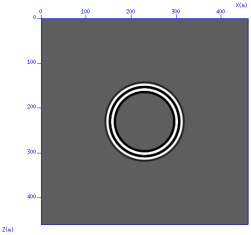
$$\vec{J} = -Q \nabla P \qquad (14)$$

Wave propagation angle

$$\theta = \arctan\left(\frac{J_z}{J_x}\right) . \qquad (15)$$

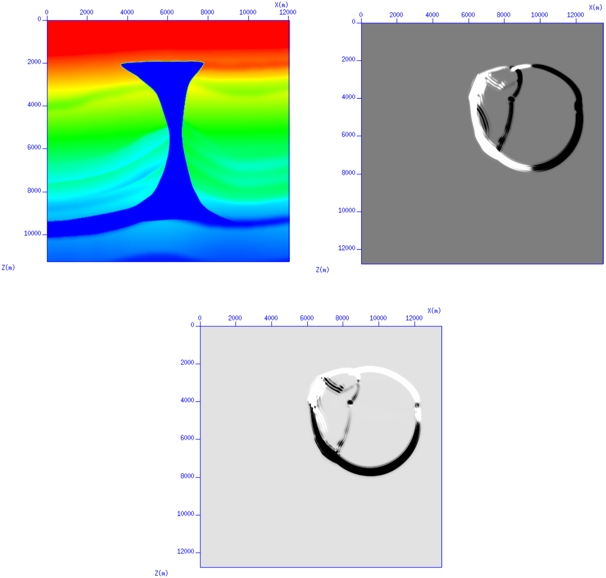




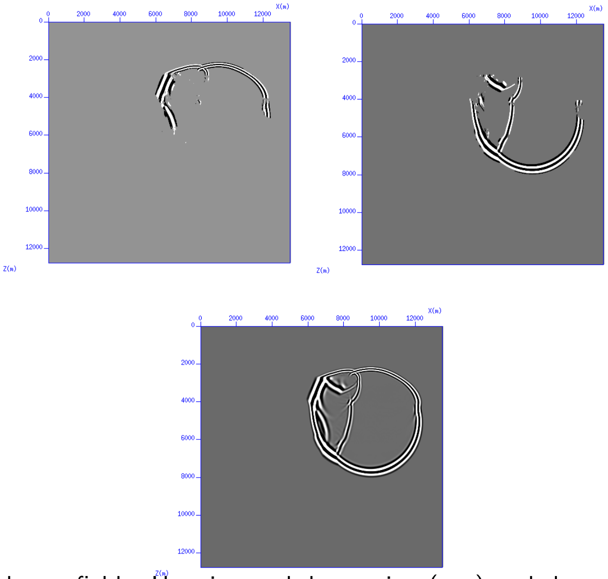


Separated wavefields: Upgoing, downgoing and the original wavefield (left to right)

Poynting vector applications

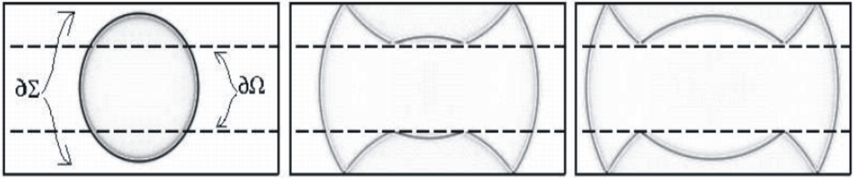


BP Model and snapshot of J_x (top) and snapshot of J_z (botton)



Separated wavefields: Upgoing and downgoing (top) and the original wavefield (botton)

Qreverse - Boundary condition problem



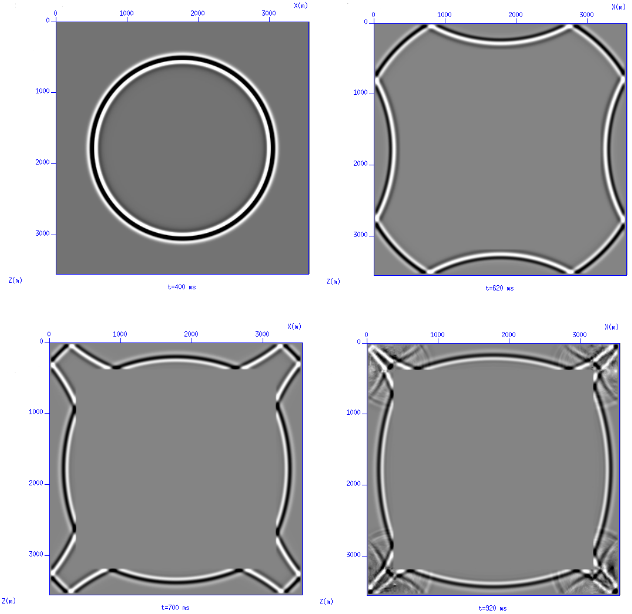
Transparent boundary when the energy leaves $\partial\Omega$ and reflective boundary when the energy returns to $\partial\Omega$ (Bonomi e Enrico, 2001).

Qreverse (Bonomi e Enrico, 2001)

$$\text{For (x,y,z)} \in \partial\Omega \quad \text{where} \quad \vec{J}.\hat{n} < 0 \tag{16}$$

$$\text{Do} \quad Q(x,y,z,t) \leftarrow -Q(x,y,z,t)\,. \tag{17}$$

Qreverse - Snapshots at different instants



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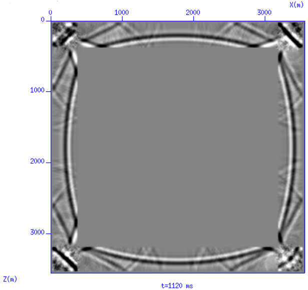
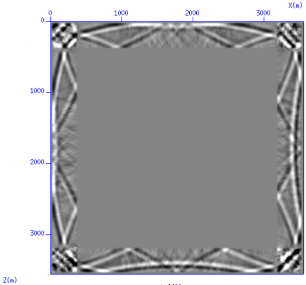
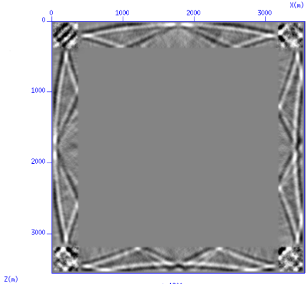


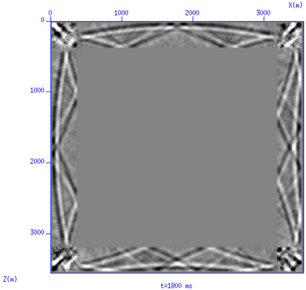
t=1120 ms



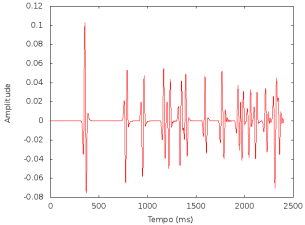
t=1400 ms



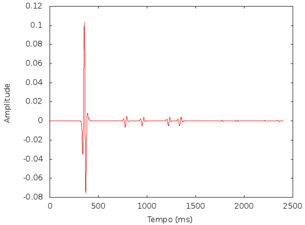
t=1600 ms



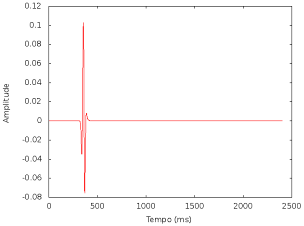
t=1800 ms



(a)



(b)



(c)

Seismic trace without qreverse (a); with taper (b) and qreverse (c)

Reverse time migration (RTM)

- In RTM, the cross-correlation imaging condition, which is given by:

$$I_{cc}(\mathbf{x}) = \int P_F(\mathbf{x}, t)\, P_B(\mathbf{x}, t)\, dt \tag{18}$$

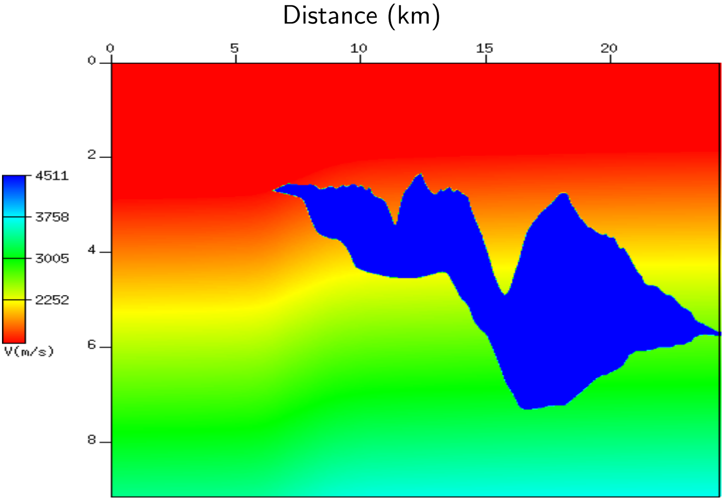
is used in practice and is often preferable due to stability reasons.

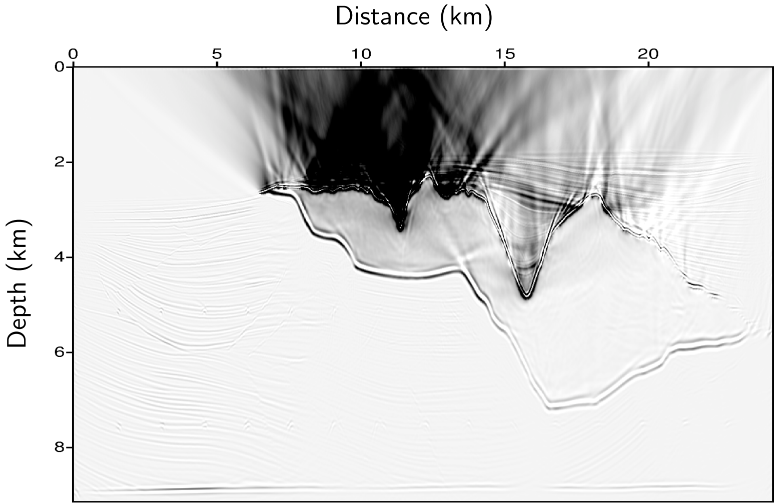
- Imaging condition proposed by Bulcão et al. (2007):

$$I_c(\mathbf{x}) = \sum_{t=0}^{t_f} P_{F_d}(\mathbf{x}, t)\, P_{B_d}(\mathbf{x}, t) \tag{19}$$

- To avoid the cross-correlation of the downgoing waves with the upgoing waves, which is the cause of the low frequency noise.

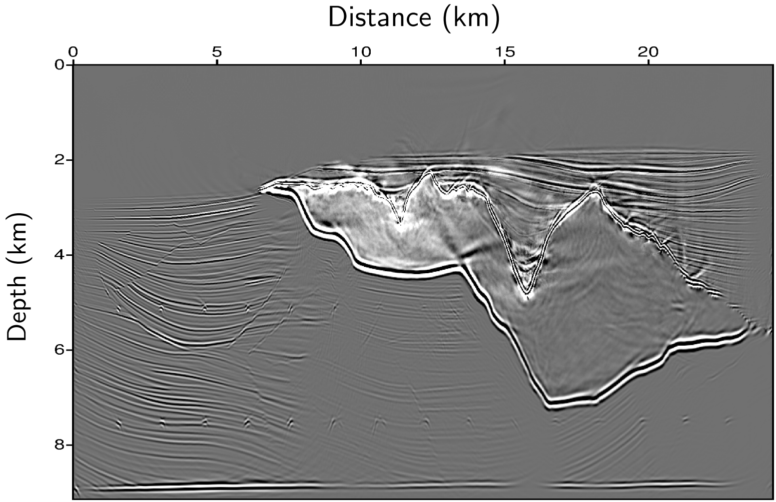
Sigsbee2A velocity model



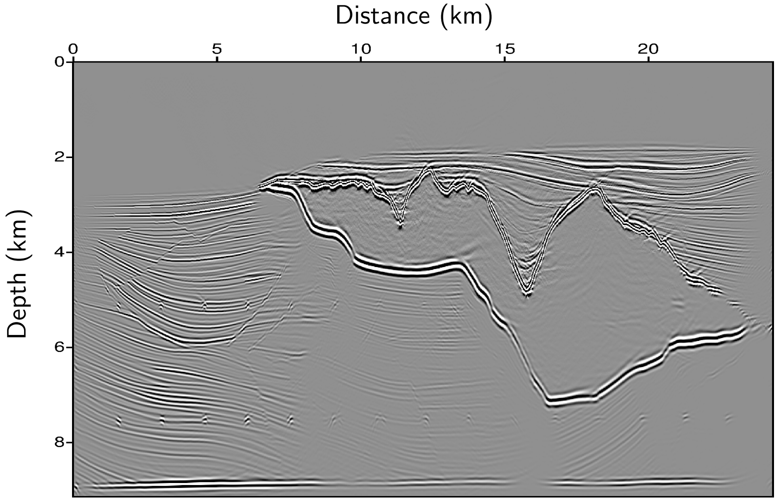


Migration result of the Sigsbee2A dataset - No filtering

Pre-stack reverse time migration - Sigsbee2A model



Migration result of the Sigsbee2A dataset - Using downgoing source and downgoing receivers parts.



Migration result of the Sigsbee2A dataset - downgoing parts plus high-pass filtering.

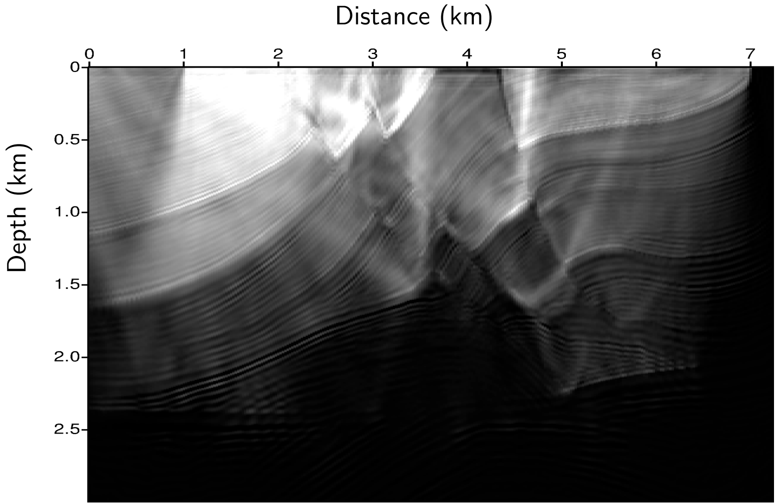
- Cross-correlation imaging condition:

$$I_{cc}(\mathbf{x}) = \int P_F(\mathbf{x}, t)\ P_B(\mathbf{x}, t)\, dt$$

- Based on the relationship between inversion and imaging (Luo et al., 2009; Zhou et al., 2009; Whitmore and Crawley, 2012):

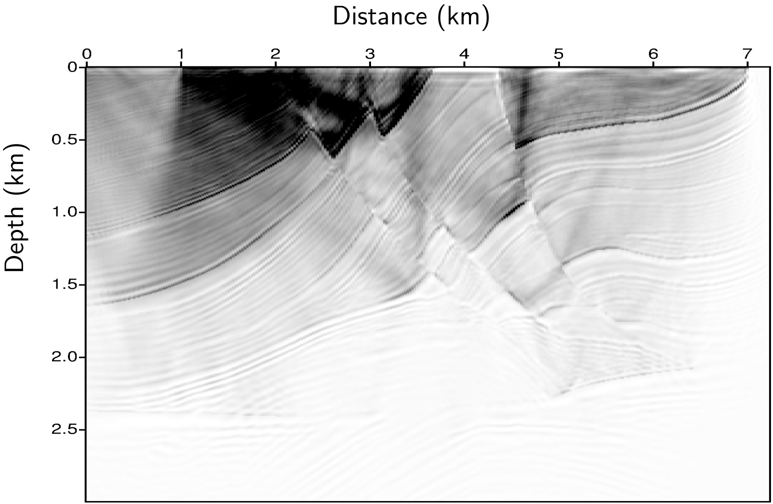
$$\begin{aligned} I_{ii}(\mathbf{x}) &= \frac{1}{v^2(\mathbf{x})} \int \frac{\partial}{\partial t} P_F(\mathbf{x}, t)\ \frac{\partial}{\partial t} P_B(\mathbf{x}, t)\ dt \\ &+ \int \nabla P_F(\mathbf{x}, t) . \nabla P_B(\mathbf{x}, t)\ dt \end{aligned} \tag{20}$$

Pre-stack RTM result - Marmousi model



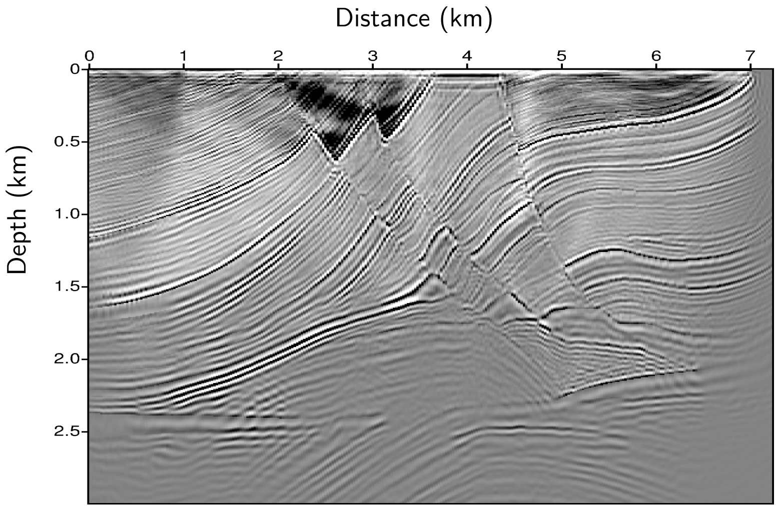
Gradient image condition.

Pre-stack RTM result - Marmousi model



Time derivative image condition.

Pre-stack RTM result - Marmousi model

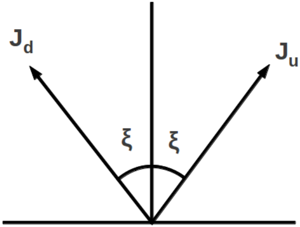


Inverse scattering image condition

Reflection angle:

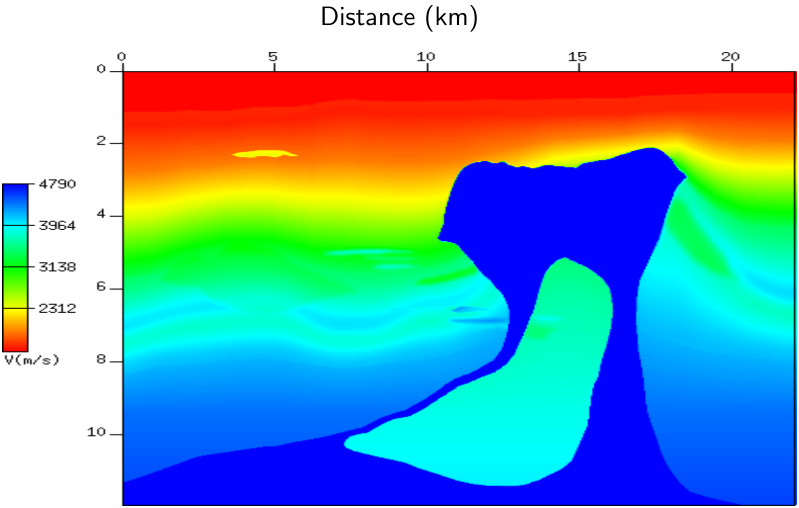
$$\cos(\psi) = \frac{\vec{J_d}.\vec{J_u}}{|\vec{J_d}||\vec{J_u}|} \; . \qquad (21)$$

$$\psi = 2\xi \quad (\xi\text{- reflection angle}) \; . \qquad (22)$$

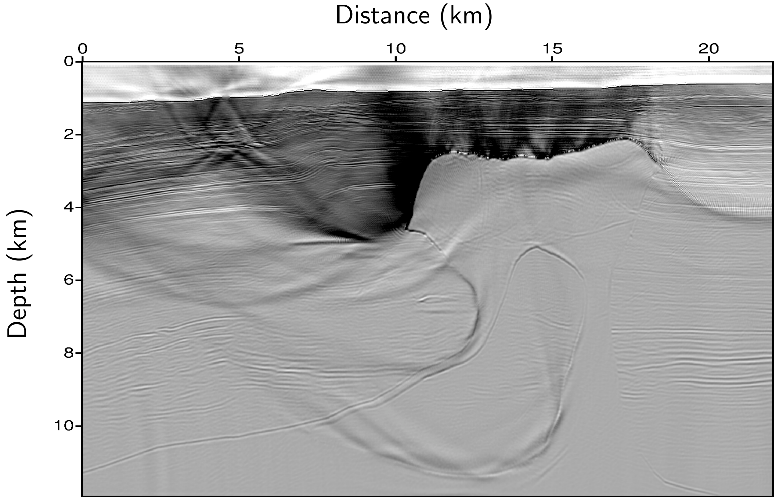


Source Poynting vector ($\vec{J_d}$) and receiver Poynting vector ($\vec{J_u}$).

2004 BP velocity model

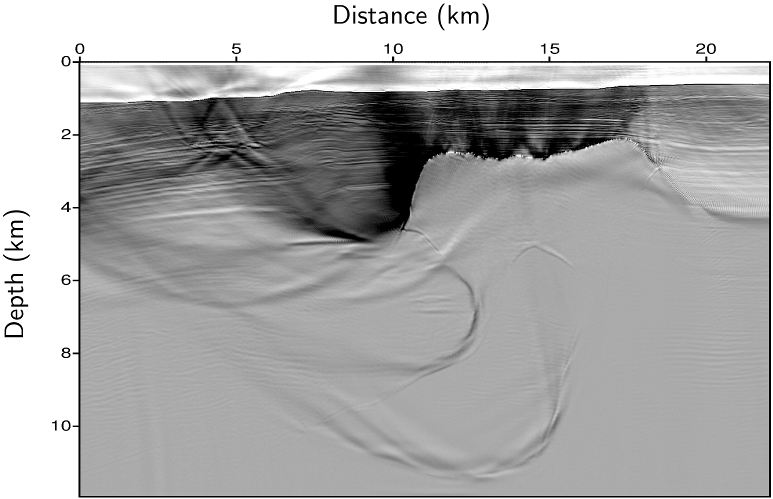


Pres-stack RTM using CIGs



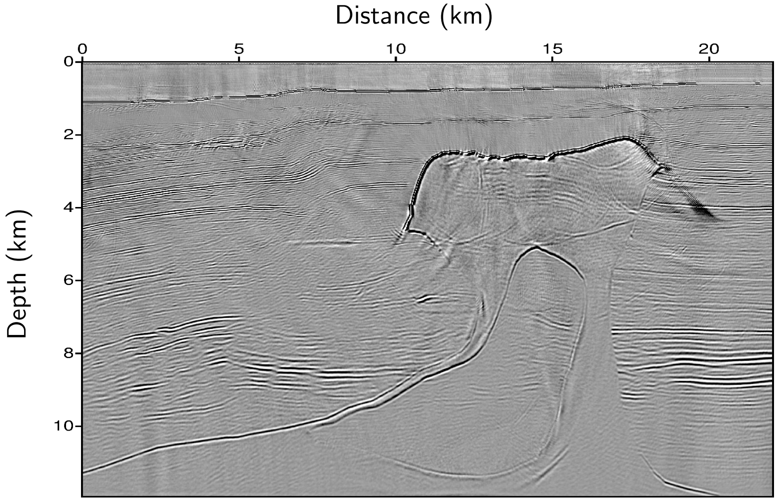
Migration result of the 2004 BP 2D dataset using the CIGs reflection angles from 0° to 90°.

Pre-stack RTM using CIGs



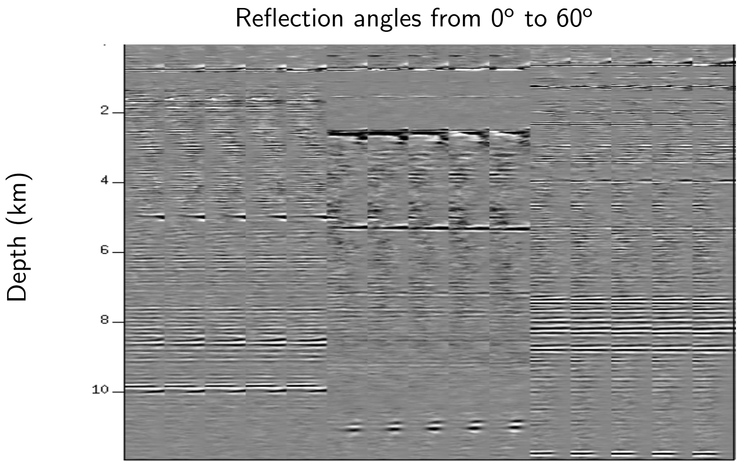
Migration result of the 2004 BP 2D dataset using the CIGs reflection angles from 61° to 90°.

Pre-stack RTM using CIGs



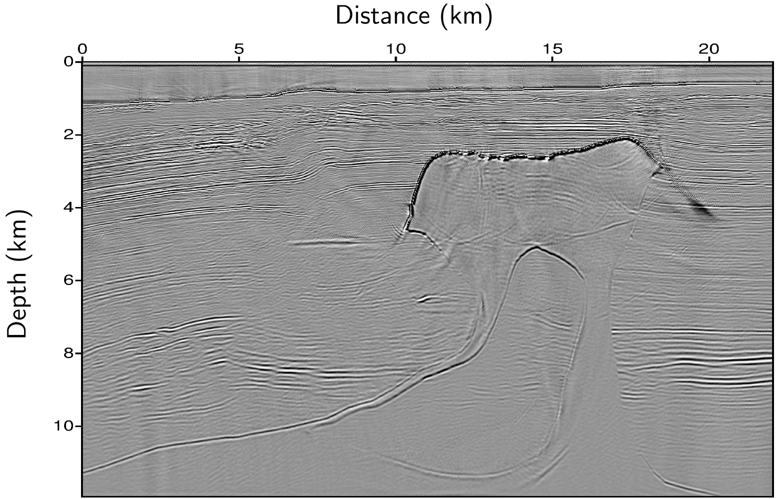
Migration result of the 2004 BP 2D dataset using the CIGs reflection angles from 0° to 60°.

CIGs in the reflection angle domain from the 2004 BP dataset



Some common image gathers from the 2004 BP dataset in the reflection angle domain.

Pre-stack RTM using CIG's - 2004 BP 2D



Final migration result obtained by summing the CIGs reflection angles from 0° to 60° and the CIGs reflection angles from 61° to 90° after high-pass filtering.

Conclusion

The new symplectic numerical scheme combined with REM proved to be a good alternative to the following applications:

- Reverse time migration
- Vector Poynting computation;
- Wavefield separation;
- Boundary condition problem;
- Common imaging gathers (CIGs);
- RTM low frequency noise attenuation.

Acknowledgements

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