An acoustic wave equation for pure P wave in 2D TTI media

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Vertical Transversely Isotropic (VTI) and Tilted Transversely Isotropic (TTI)
Tilted Transversely Isotropic (TTI)
Introduction

Global (vertical) symmetry assumption

Local (tilted) symmetry assumption (more realistic)
Motivation

VTI RTM image - The sub-salt image is incoherent and defocused.

TTI RTM image - Continuous subsalt sediments and clear terminations.

(From Huang et al., 2009)
The 3D TTI coupled equations (Fletcher, 2008; Zhang and Zhang, 2008) \((v_s = 0.0)\)

\[
\begin{align*}
\frac{1}{v_{p0}^2} \frac{\partial^2 p}{\partial t^2} &= (1 + 2\delta)H_2(p + q) + H_1 p \\
\frac{1}{v_{p0}^2} \frac{\partial^2 q}{\partial t^2} &= 2(\epsilon - \delta)H_2(p + q)
\end{align*}
\]

\[
\begin{align*}
H_1 &= [\sin \theta \cos \phi \partial_x + \sin \theta \sin \phi \partial_y + \cos \theta \partial_z]^2 \\
H_2 &= (\partial_x^2 + \partial_y^2 + \partial_z^2) - H_1
\end{align*}
\]

where \(p\) is the pressure wavefield, \(q\) is an introduced auxiliary wavefield, \(\epsilon\) and \(\delta\) are Thomson’s parameter; \(\theta\) and \(\phi\) are the dip angle and azimuth angle of the symmetry axis.
BP 2007 TTI model - parameters
BP 2007 TTI model

Dataset Benchmark - Modeling
Forward Modeling Simulation (unstable)
Forward Modeling Simulation (unstable)
Forward Modeling Simulation (unstable)
Forward Modeling Simulation (unstable)
Forward Modeling Simulation (unstable)
Title Angle Variation

Distance (km)

Depth (km)

Title angle $\theta$

$\text{grad}(\theta)$
Snapshots

(a) Unstable TTI snapshot at t=8 sec

(a) Stable TTI snapshot at t=8 sec
RTM images - Old methods

Isotropic RTM Image

TTI RTM Image

Depth (km)

Distance (km)
RTM images - Old methods

Isotropic RTM

TTI RTM
The equations of P and SV wave phase velocity gives (Pestana et al., 2011 - 12th CISBGf)

\[
\begin{align*}
\omega^2 &= v_{p0}^2 \left[ (1 + 2\epsilon) k_r^2 + k_z^2 - \frac{2(\epsilon-\delta) k_r^2 k_z^2}{k_z^2 + F k_r^2} \right] \\
\omega^2 &= v_{p0}^2 \left[ \frac{v_{s0}^2}{v_{p0}^2} (k_r^2 + k_z^2) + \frac{2(\epsilon-\delta) k_r^2 k_z^2}{k_z^2 + F k_r^2} \right]
\end{align*}
\]

where \( F = 1 + \frac{2\epsilon}{f} \). For simplicity, we proceed with a choice \( F = 1 \).

Equations hold for TI media with a vertical symmetry axis (VTI).
Decoupled wave equations equation for TTI media

Dispersion relations for TTI media with arbitrary orientation of symmetry axis can be deduced from VTI equations through a variable change (3D rotation).

The wavenumber operators in the rotated coordinates system write

\[
\begin{bmatrix}
\hat{k}_x \\
\hat{k}_y \\
\hat{k}_z \\
\end{bmatrix} =
\begin{bmatrix}
\cos \theta \cos \phi & \cos \theta \sin \phi & \sin \theta \\
-\sin \phi & \cos \phi & 0 \\
-\sin \theta \cos \phi & -\sin \theta \sin \phi & \cos \theta \\
\end{bmatrix}
\begin{bmatrix}
k_x \\
k_y \\
k_z \\
\end{bmatrix}
\]

Then we have:

\[
\begin{cases}
\hat{k}_r^2 = k_r^2 - \sin^2 \theta (\cos^2 \phi k_x^2 + \sin^2 \phi k_y^2 - k_z^2 + \sin 2\phi k_x k_y) \\
+ \sin 2\theta (\cos \phi k_x k_z + \sin \phi k_y k_z) \\

\hat{k}_z^2 = k_z^2 - \sin^2 \theta (\cos^2 \phi k_x^2 + \sin^2 \phi k_y^2 - k_z^2 + \sin 2\phi k_x k_y) \\
- \sin 2\theta (\cos \phi k_x k_z + \sin \phi k_y k_z)
\end{cases}
\]
Time-wavenumber equations for TTI media

2-D case version for P wave:

\[
\frac{1}{v_p^2} \frac{\partial^2 P}{\partial t^2} = - \left\{ k_x^2 + k_z^2 \right. \\
+ (2\epsilon \cos^4 \theta + 2\delta \sin^2 \theta \cos^2 \theta) \frac{k_x^4}{k_x^2 + k_z^2} + (2\epsilon \sin^4 \theta + 2\delta \sin^2 \theta \cos^2 \theta) \frac{k_z^4}{k_x^2 + k_z^2} \\
+ (-4\epsilon \sin 2\theta \cos^2 \theta + \delta \sin 4\theta) \frac{k_x^3 k_z}{k_x^2 + k_z^2} + (-4\epsilon \sin 2\theta \sin^2 \theta - \delta \sin 4\theta) \frac{k_x k_z^3}{k_x^2 + k_z^2} \\
+ (3\epsilon \sin^2 2\theta + \delta \cos^2 2\theta + \delta \cos 4\delta) \frac{k_x k_z^2}{k_x^2 + k_z^2} \left\} P
\]

and SV wave:

\[
\frac{1}{v_p^2} \frac{\partial^2 P_{SV}}{\partial t^2} = - \left\{ \frac{v_{p0}^2}{v_{SO}^2} (k_x^2 + k_z^2) + (\epsilon - \delta) \right\} \left\{ 2 \sin^2 \theta \cos^2 \theta \frac{k_x^4}{k_x^2 + k_z^2} \\
+ 2 \sin^2 \theta \cos^2 \theta \frac{k_z^4}{k_x^2 + k_z^2} + \sin 4\theta \frac{k_x^3 k_z}{k_x^2 + k_z^2} + (- \sin 4\theta) \frac{k_x k_z^3}{k_x^2 + k_z^2} \\
+ (\cos^2 2\theta + \cos 4\theta) \frac{k_x k_z^2}{k_x^2 + k_z^2} \right\} P_{SV}
\]
The solution of the P pure wave equation can be written as (Pestana and Stoffa, 2010)

\[ p(t + \Delta t) = -p(t - \Delta t) + 2 \cos(L\Delta t)p(t) \]

where the pseudo-differential operator is defined as

\[
-L^2 = - \left\{ k_x^2 + k_z^2 \\
+ (2\epsilon \cos^4 \theta + 2\delta \sin^2 \theta \cos^2 \theta) \frac{k_x^4}{k_x^2 + k_z^2} + (2\epsilon \sin^4 \theta + 2\delta \sin^2 \theta \cos^2 \theta) \frac{k_z^4}{k_x^2 + k_z^2} \\
+ (-4\epsilon \sin 2\theta \cos^2 \theta + \delta \sin 4\theta) \frac{k_x^3 k_z}{k_x^2 + k_z^2} + (-4\epsilon \sin 2\theta \sin^2 \theta - \delta \sin 4\theta) \frac{k_x k_z^3}{k_x^2 + k_z^2} \\
+ (3\epsilon \sin^2 2\theta + \delta \cos^2 2\theta + \delta \cos 4\delta) \frac{k_x^2 k_z^2}{k_x^2 + k_z^2} \right\} P
\]

The cosine function is approximated by

\[
\cos(L\Delta t) = \sum_{k=0}^{M} C_{2k} J_{2k}(R\Delta t) Q_{2k}(iL/R) \quad M > R\Delta t
\]

For anisotropic the value of R for 2D case is given by

\[ R = \pi v_{\text{max}} (1 + |\epsilon|_{\text{max}}) \sqrt{1/\Delta x^2 + 1/\Delta z^2} \]
$V_{p0} = 3000 \text{m/s}; \epsilon = 0.24; \delta = 0.0$ and $\theta = 45^0$

TTI coupled equations $V_{s0} = 0.0$ (a); non zero $V_{s0}$ wave velocity (b) Pure P wave (c) and SV (d)
Anisotropic parameters - 2D wedge model
Wavefield snapshots - 2D wedge model

Wedge model (a); TTI coupled equations $V_{s0} = 0.0$ (b); non zero $V_{s0}$ wave velocity (c) Pure P wave (d).
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Wavefield snapshots - 2D wedge model

Wedge model (a); TTI coupled equations $V_{s0} = 0.0$ (b); non zero $V_{s0}$ wave velocity (c) Pure P wave (d).
2D BP TTI model (partial region)
Wavefield snapshots in the 2D BP TTI model

Gradient of dip angle model (a); TTI coupled equations $V_{s0} = 0.0$ (b); with a finite $V_{s0}$ wave velocity (c) Pure P wave (d).
RTM images - New method

VTI REM of the partial BP model
RTM images - New method

TTI REM of the partial BP model
RTM images - Zoom

Vp

VTI RTM

TTI RTM
Conclusions

- We present an approach for modeling and migration in an acoustic TTI media using decoupled P wave and SV wave equations.

- Compared with TTI coupled wave equations published in the geophysics literature, the proposed decoupled equations are stable.

- To avoid numerical dispersion and produce high quality images, the rapid expansion method (REM) and pseudo-spectral method are employed for numerical implementation.

- To make this RTM computation possible high speed and parallel computers are needed. (For examples, GPU clusters)
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