An acoustic wave equation for pure P wave in 2D TTI media

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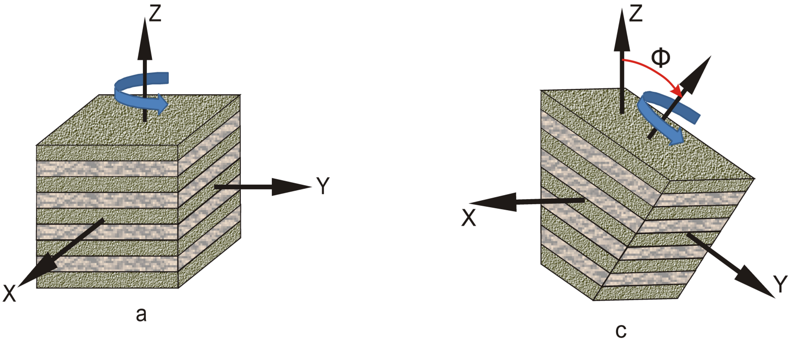
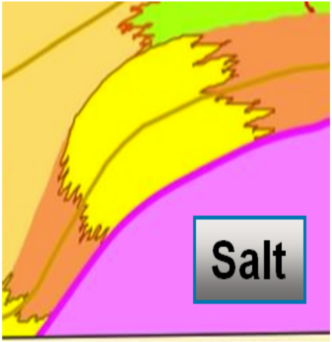
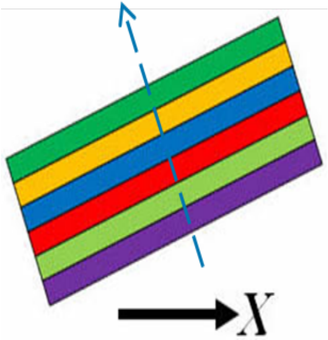


Vertical Transversely Isotropic (VTI) and Tilted Transversely Isotropic (TTI)

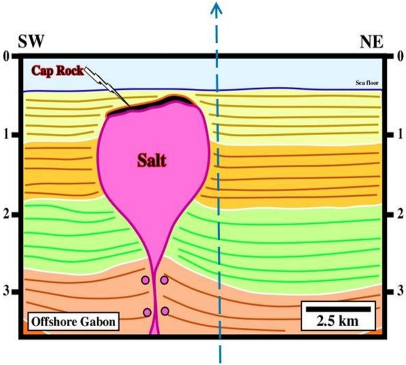
Introduction



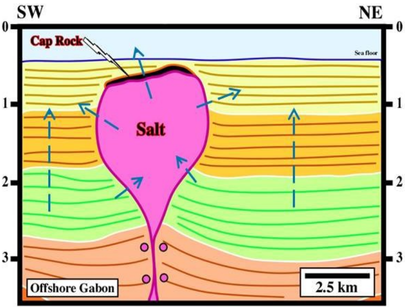


Tilted Transversely Isotropic (TTI)

Introduction

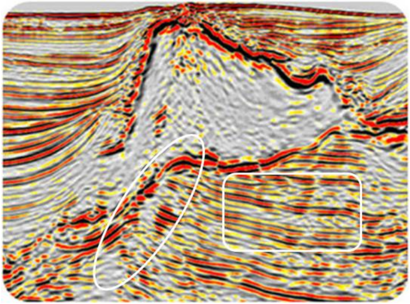


Global (vertical) symmetry assumption

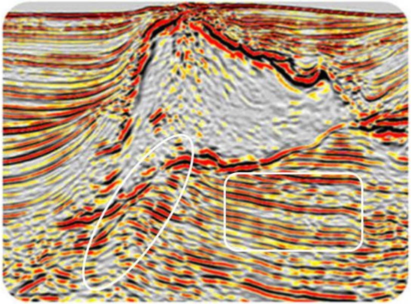


Local (tilted) symmetry assumption (more realistic)

Motivation



VTI RTM image - The sub-salt image is incoherent and defocused.



TTI RTM image - Continuous subsalt sediments and clear terminations.

(From Huang et al., 2009)

The 3D TTI coupled equations - Current practice

The 3D TTI coupled equations (Fletcher, 2008; Zhang and Zhang, 2008) ($v_s = 0.0$)

$$\begin{cases} \frac{1}{v_{p0}^2}\frac{\partial^2 p}{\partial t^2} & = (1+2\delta)H_2(p+q) + H_1 p \\ \frac{1}{v_{p0}^2}\frac{\partial^2 q}{\partial t^2} & = 2(\epsilon-\delta)H_2(p+q) \end{cases}$$

$$\begin{cases} H_1 & = [\sin\theta\cos\phi\partial_x + \sin\theta\sin\phi\partial_y + \cos\theta\partial_z]^2 \\ H_2 & = (\partial_x^2 + \partial_y^2 + \partial_z^2) - H_1 \end{cases}$$

where p is the pressure wavefield, q is an introduced auxiliary wavefield, ϵ and δ are Thomson's parameter; θ and ϕ are the dip angle and azimuth angle of the symmetry axis.

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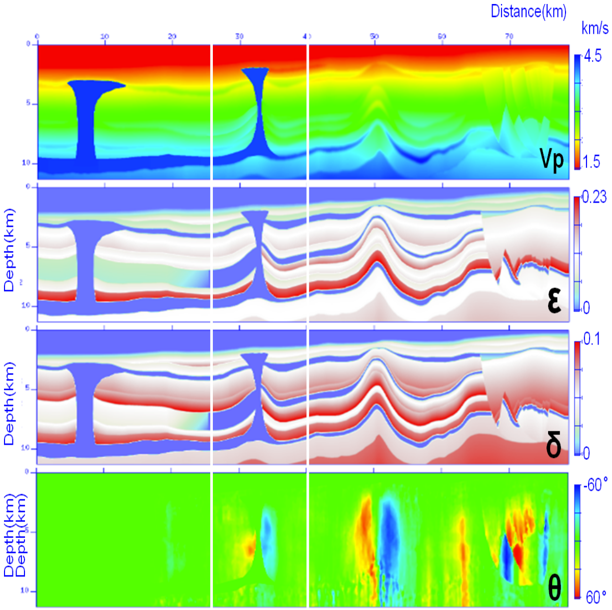
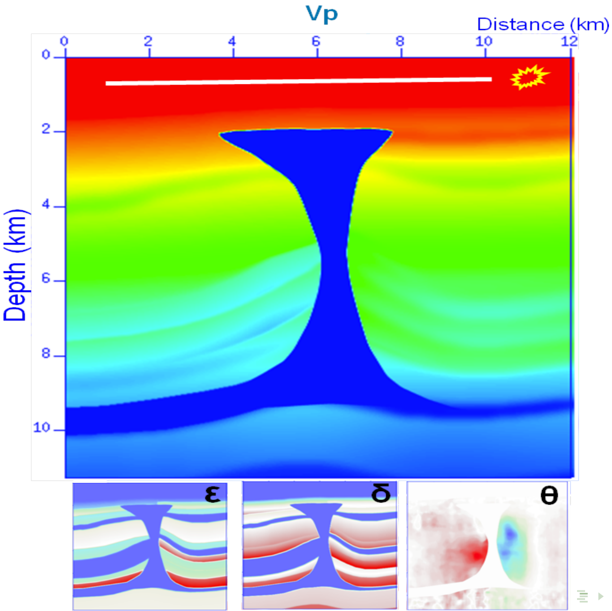
BP 2007 TTI model - parameters

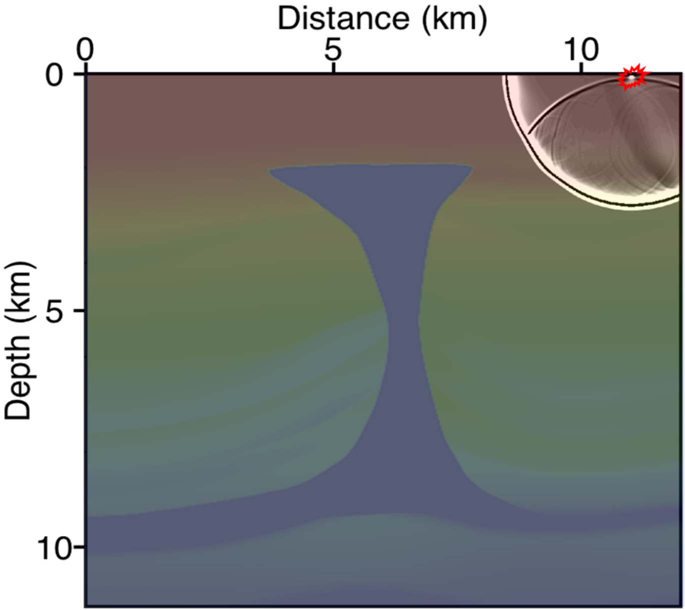


BP 2007 TTI model

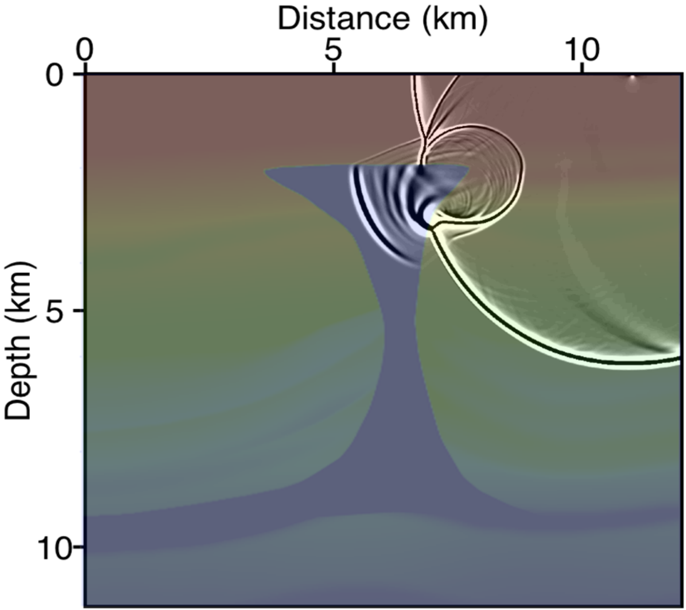
Dataset Benchmark - Modeling



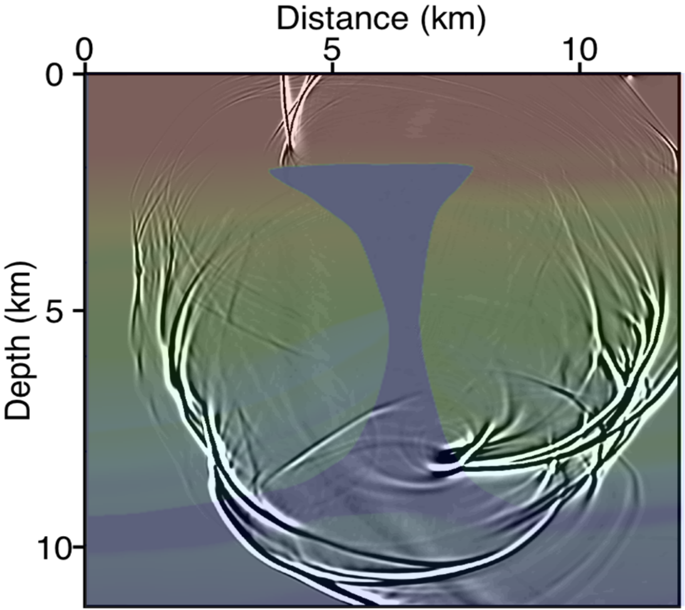
Forward Modeling Simulation (unstable)



Forward Modeling Simulation (unstable)



Forward Modeling Simulation (unstable)



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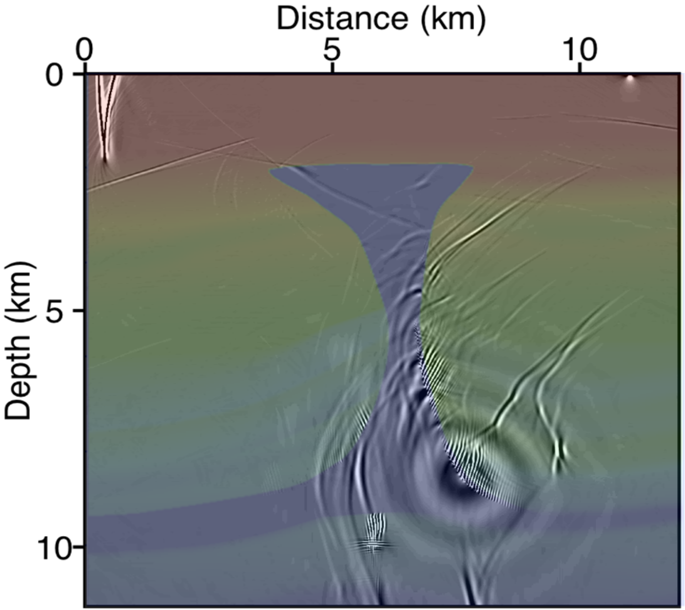
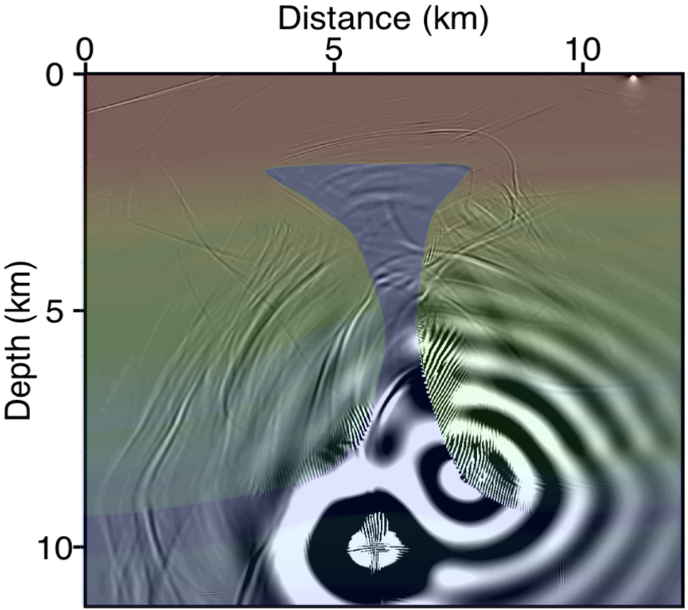
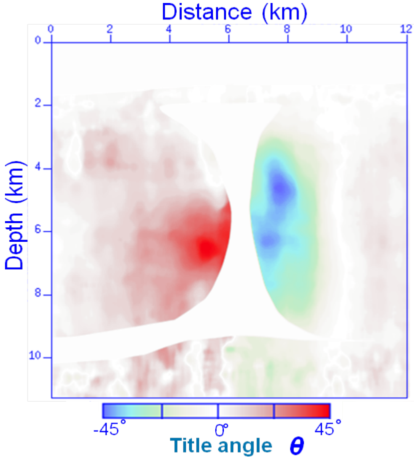
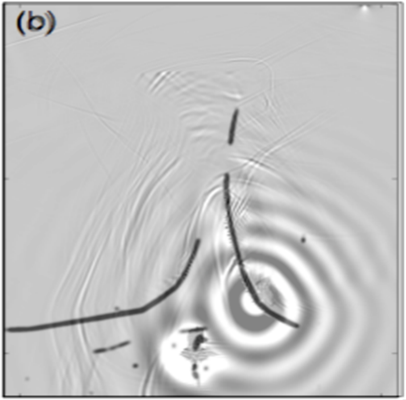


Forward Modeling Simulation (unstable)



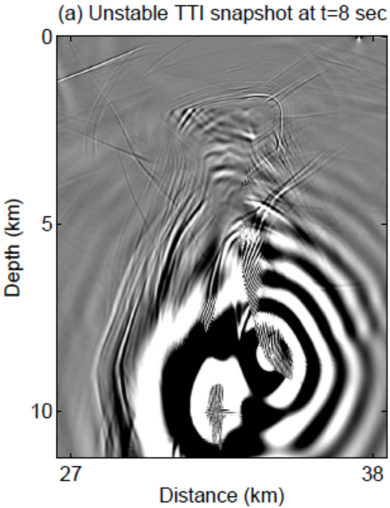
Tilte Angle Variation

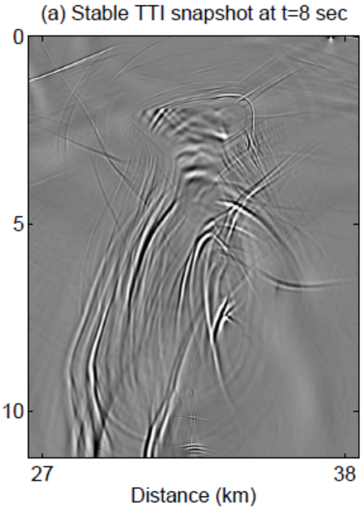




grad(θ)

Snapshots





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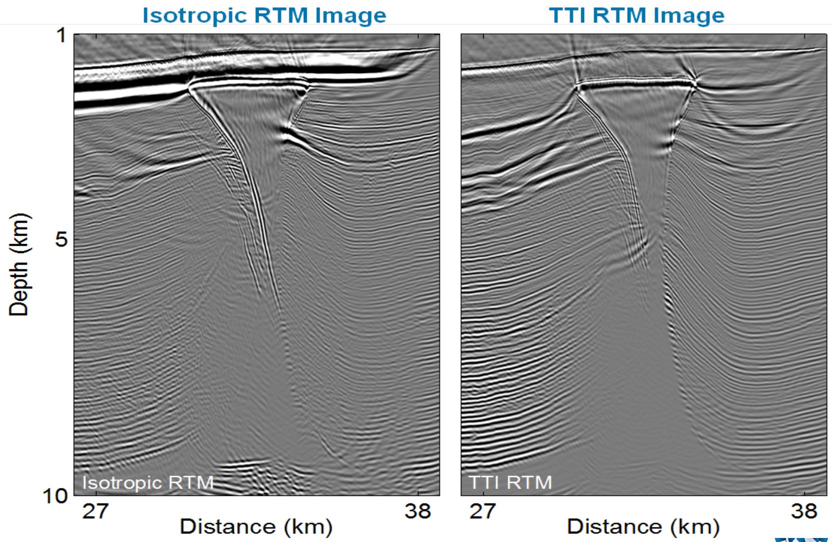
RTM images - Old methods



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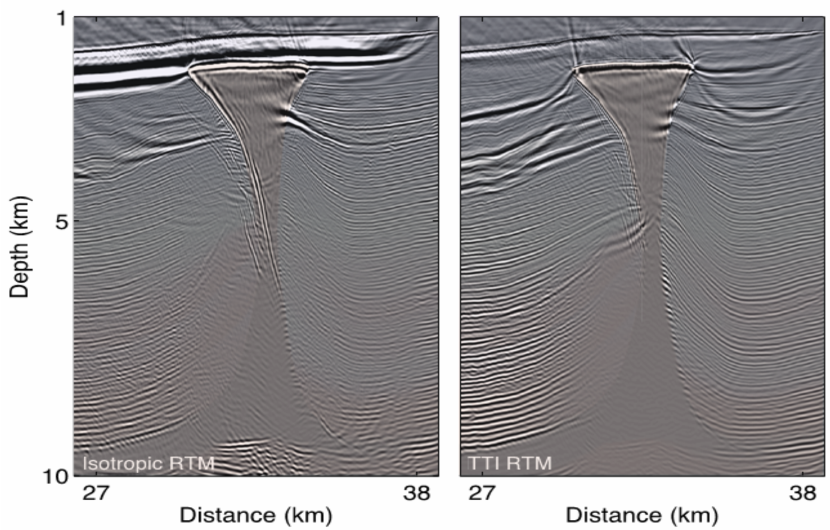


Decoupled wave equations for VTI

The equations of P and SV wave phase velocity gives (Pestana et al., 2011 - 12th CISBGf)

$$\begin{cases} \omega^2 & = v_{p0}^2 \left[(1 + 2\epsilon)\, k_r^2 + k_z^2 - \frac{2(\epsilon - \delta)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \right] \\ \omega^2 & = v_{p0}^2 \left[\frac{v_{s0}^2}{v_{p0}^2} \,(k_r^2 + k_z^2) + \frac{2(\epsilon - \delta)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \right] \end{cases}$$

where $F = 1 + \frac{2\epsilon}{f}$. For simplicity, we proceed with a choice $F = 1$.

Equations hold for TI media with a vertical symmetry axis (VTI).

Decoupled wave equations equation for TTI media

Dispersion relations for TTI media with arbitrary orientation of symmetry axis can be deduced from VTI equations through a variable change (3D rotation).

The wavenumber operators in the rotated coordinates system write

$$\begin{bmatrix} \hat{k}_x \\ \hat{k}_y \\ \hat{k}_z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & \sin\theta \\ -\sin\phi & \cos\phi & 0 \\ -\sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta \end{bmatrix} \begin{bmatrix} k_x \\ k_y \\ k_z \end{bmatrix}$$

Then we have:

$$\begin{cases} \hat{k}_r^2 &= k_r^2 - \sin^2\theta(\cos^2\phi k_x^2 + \sin^2\phi k_y^2 - k_z^2 + \sin 2\phi k_x k_y) \\ & + \sin 2\theta(\cos\phi k_x k_z + \sin\phi k_y k_z) \\ \hat{k}_z^2 &= k_z^2 - \sin^2\theta(\cos^2\phi k_x^2 + \sin^2\phi k_y^2 - k_z^2 + \sin 2\phi k_x k_y) \\ & - \sin 2\theta(\cos\phi k_x k_z + \sin\phi k_y k_z) \end{cases}$$

Time-wavenumber equations for TTI media

2-D case version for P wave:

$$\left\{ \begin{aligned} \frac{1}{v_{p0}^2}\frac{\partial^2 P}{\partial t^2} &= -\Big\{ k_x^2 + k_z^2 \\ &+(2\epsilon\, cos^4\theta + 2\delta sin^2\theta \cos^2\theta)\frac{k_x^4}{k_x^2+k_z^2} + (2\epsilon \sin^4\theta + 2\delta\sin^2\theta\cos^2\theta)\frac{k_z^4}{k_x^2+k_z^2} \\ &+(-4\epsilon\sin 2\theta\cos^2\theta + \delta\sin 4\theta)\frac{k_x^3\, k_z}{k_x^2+k_z^2} + (-4\epsilon\sin 2\theta\sin^2\theta - \delta\sin 4\theta)\frac{k_x\, k_z^3}{k_x^2+k_z^2} \\ &+(3\epsilon\sin^2 2\theta + \delta\cos^2 2\theta + \delta\cos 4\delta)\,\frac{k_x^2\, k_z^2}{k_x^2+k_z^2}\Big\} P \end{aligned} \right.$$

and SV wave:

$$\left\{ \begin{aligned} \frac{1}{v_{p0}^2}\frac{\partial^2 P_{SV}}{\partial t^2} &= -\Big\{ \frac{v_{p0}^2}{v_{s0}^2}(k_x^2 + k_z^2) + (\epsilon - \delta)\{2\sin^2\theta\cos^2\theta\frac{k_x^4}{k_x^2+k_z^2} \\ &+2\sin^2\theta\cos^2\theta\frac{k_z^4}{k_x^2+k_z^2} + \sin 4\theta\frac{k_x^3\, k_z}{k_x^2+k_z^2} + (-\sin 4\theta)\frac{k_x\, k_z^3}{k_x^2+k_z^2} \\ &+(\cos^2 2\theta + \cos 4\theta)\,\frac{k_x^2\, k_z^2}{k_x^2+k_z^2}\}\Big\} P_{SV} \end{aligned} \right.$$

Rapid expansion method

The solution of the P pure wave equation can be written as (Pestana and Stoffa, 2010)

$$p(t+\Delta t) = -p(t-\Delta t) + 2\cos(L\Delta t)p(t)$$

where the pseudo-differential operator is defined as

$$\begin{cases} -L^2 & = -\Big\{ k_x^2 + k_z^2 \\ & +(2\epsilon\, cos^4\theta + 2\delta sin^2\theta\cos^2\theta)\frac{k_x^4}{k_x^2+k_z^2} + (2\epsilon\sin^4\theta + 2\delta\sin^2\theta\cos^2\theta)\frac{k_z^4}{k_x^2+k_z^2} \\ & +(-4\epsilon\sin 2\theta\cos^2\theta + \delta\sin 4\theta)\frac{k_x^3\,k_z}{k_x^2+k_z^2} + (-4\epsilon\sin 2\theta\sin^2\theta - \delta\sin 4\theta)\frac{k_x\,k_z^3}{k_x^2+k_z^2} \\ & +(3\epsilon\sin^2 2\theta + \delta\cos^2 2\theta + \delta\cos 4\delta)\frac{k_x^2\,k_z^2}{k_x^2+k_z^2}\Big\} P \end{cases}$$

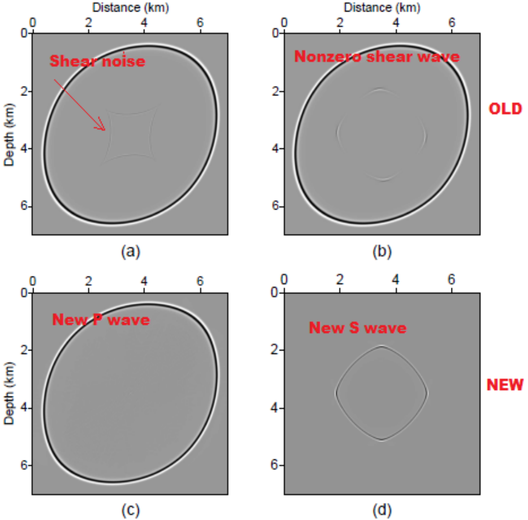
The cosine function is approximated by

$$cos(L\Delta t) = \sum_{k=0}^{M} C_{2k} J_{2k}(R\Delta t) Q_{2k}(iL/R) \qquad M > R\Delta t$$

For anisotropic the value of R for 2D case is given by $R = \pi v_{max}(1+|\epsilon|_{max})\sqrt{1/\Delta x^2 + 1/\Delta z^2}$

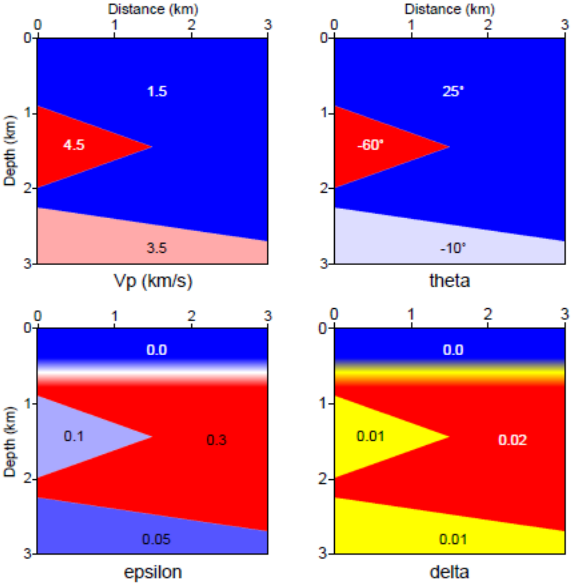
Wavefield Snapshots

$V_{p0} = 3000 m/s; \epsilon = 0.24; \delta = 0.0$ and $\theta = 45^0$

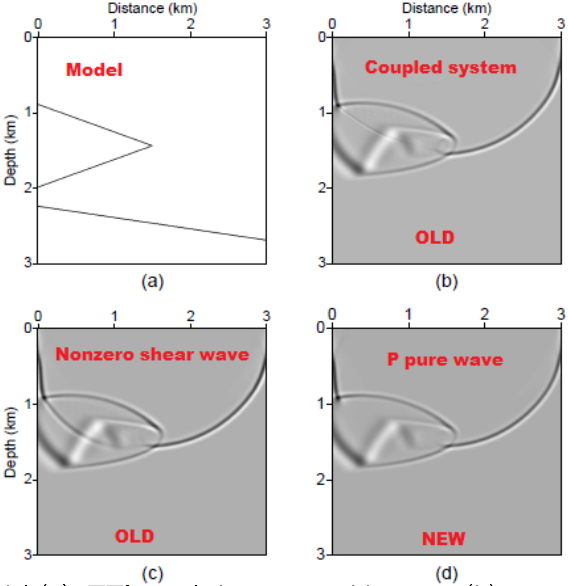


TTI coupled equations $V_{s0} = 0.0$ (a); non zero V_{s0} wave velocity (b) Pure P wave (c) and SV (d)

Anisotropic parameters - 2D wedge model

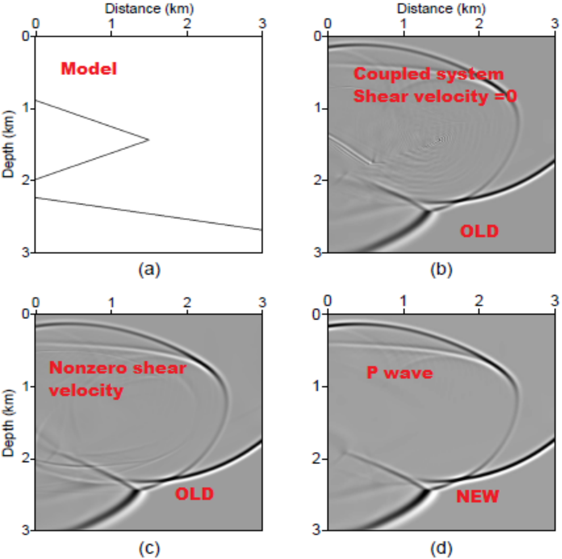


Wavefield snapshots - 2D wedge model



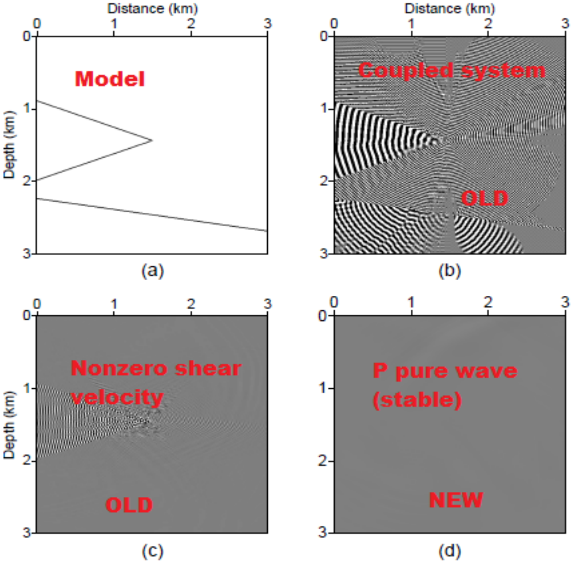
Wedge model (a); TTI coupled equations $V_{s0} = 0.0$ (b); non zero V_{s0}
wave velocity (c) Pure P wave (d).

Wavefield snapshots - 2D wedge model



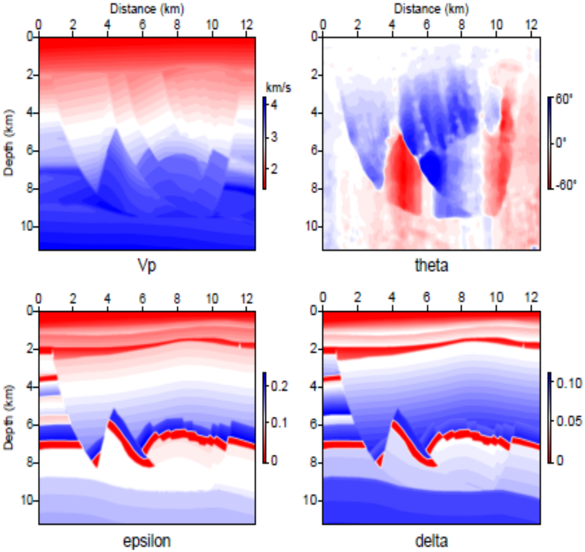
Wedge model (a); TTI coupled equations $V_{s0} = 0.0$ (b); non zero V_{s0} wave velocity (c) Pure P wave (d).

Wavefield snapshots - 2D wedge model

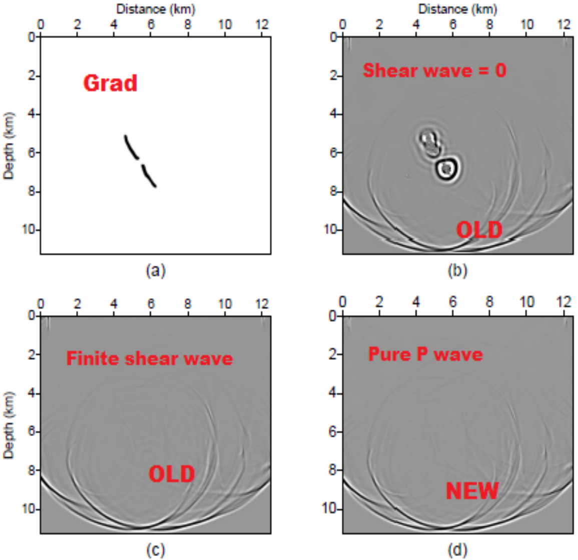


Wedge model (a); TTI coupled equations $V_{s0} = 0.0$ (b); non zero V_{s0}
wave velocity (c) Pure P wave (d).

2D BP TTI model (partial region)

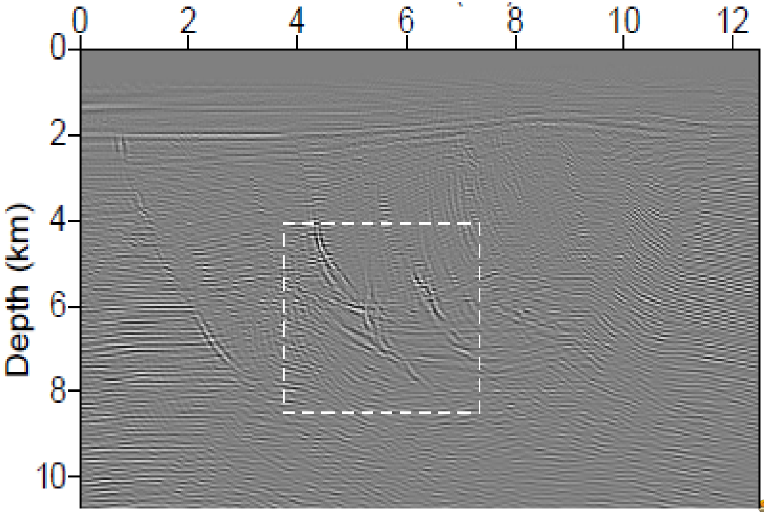


Wavefield snapshots in the 2D BP TTI model



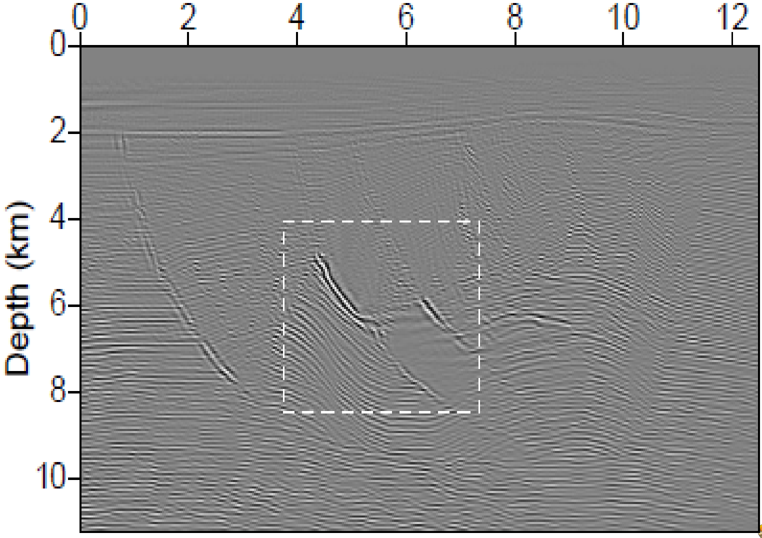
Gradient of dip angle model (a); TTI coupled equations $V_{s0} = 0.0$ (b); with a finite V_{s0} wave velocity (c) Pure P wave (d).

RTM images - New method



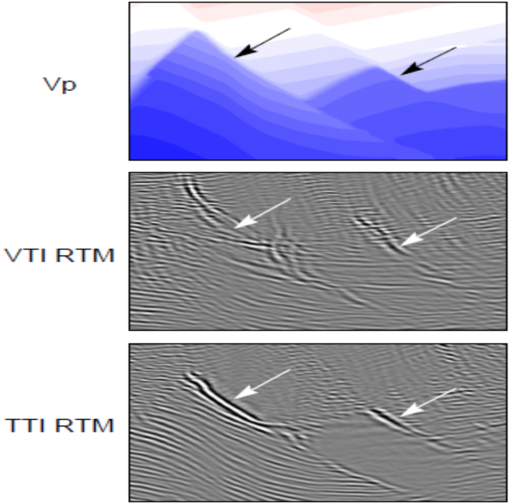
VTI REM of the partial BP model

RTM images - New method



TTI REM of the partial BP model

RTM images - Zoom



Conclusions

- We present an approach for modeling and migration in an acoustic TTI media using decoupled P wave and SV wave equations.
- Compared with TTI coupled wave equations published in the geophysics literature, the proposed decoupled equations are stable.
- To avoid numerical dispersion and produce high quality images, the rapid expansion method (REM) and pseudo-spectral method are employed for numerical implementation.
- To make this RTM computation possible high speed and parallel computers are needed. (For examples, GPU clusters)

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Acknowledgments

- The authors for funding from the King Abdullah University of Science and Technology (KAUST).

- Pestana for funding from CNPq and INCT-GP/CNPq.

- BP for making the 2007 2D TTI benchmark dataset and velocity model available.