Separate P- and SV-wave equatios for VTI media

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Outline

- Motivation
- Acoustic wave equation for VTI media
  - Dispersion relation for VTI media
  - Coupled system of second-order PDEs for VTI media
  - Decoupled P- and SV wave equations
- Rapid expansion method - REM
- Numerical results - Impulse response and VTI Hess dataset
- Conclusions
- Acknowledgments

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Motivation

- Anisotropy case:
  - Conventional isotropic methods for seismic data processing are subject to errors in transversely isotropic (TI) media.
  - Ignoring the effect of anisotropy in imaging may results in significant mispositioning of steeply reflectors (beneath or inside anisotropic structures)
  - Pseudo-acoustic wave eqaution (SV-wave noise)
- Separate P and S-waves for imaging condition - Full elastic wave has P and S waves implicited coupled - complete separation remains a subject for ongoing research.

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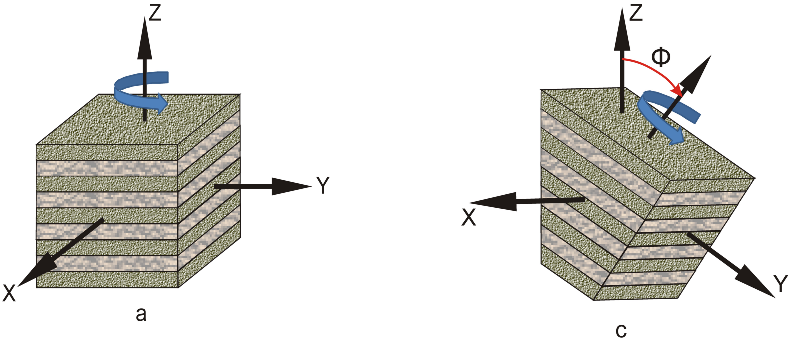
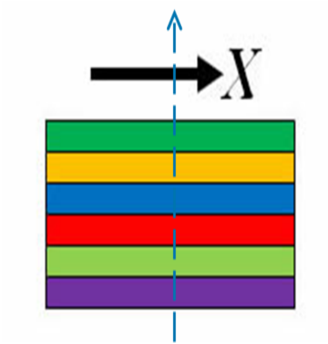
VTI and TTI media



Vertical Transversely Isotropic (VTI) and Tilted Transversely Isotropic (TTI)

VTI



Vertical Transversely Isotropic (VTI)

Wave equation in acoustic VTI media

Acoustic anisotropy is introduced by setting the shear wave velocity to zero, i.e., $v_s = 0$, along the symmetric axis (Alkhalifah, 1998).

The Dispersion relation for waves in 3D acoustic VTI media (Alkhalifah, 2000) is given by:

$$\omega^4 - \left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \omega^2 - v_{po}^2 (v_n^2 - v_h^2)\, k_r^2\, k_z^2 = 0 \qquad (1)$$

- k_x, k_y and k_z are wavenumbers in the x, y and z directions, $k_r^2 = k_x^2 + k_y^2$
- ω is the angular frequency; v_{po} is the vertical P velocity.
- $v_n = v_{po}\sqrt{1+2\delta}$ is the P-wave normal moveout (NMO) velocity;
- $v_h = v_{po}\sqrt{1+2\epsilon}$ is the horizontal P velocity;
- δ and ϵ are anisotropic parameters - Thomsen (1986).

Wave equation - Du et at. (2008)

Introducing the new auxiliary function

$$q(\omega, k_x, k_y, k_z) = \frac{\omega^2 + (v_n^2 - v_h^2)\left(k_x^2 + k_y^2\right)}{\omega^2} p(\omega, k_x, k_y, k_z) \quad (2)$$

Now the equation 1 can be written as

$$\begin{aligned} \omega^2 p(\omega, k_x, k_y, k_z) &= v_h^2 \left(k_x^2 + k_y^2\right) p(\omega, k_x, k_y, k_z) \\ &+ v_{po}^2 k_z^2 q(\omega, k_x, k_y, k_z) \end{aligned} \quad (3)$$

Wave equation - Du et at. (2008)

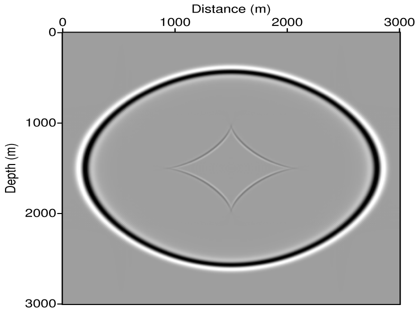
Applying an inverse Fourier to both sides of the previous two equations, we obtain the following pseudo-acoustic VTI system of equations

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} &= v_h^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_{po}^2 \frac{\partial^2 q}{\partial z^2} \\ \frac{\partial^2 q}{\partial t^2} &= v_n^2 \left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) + v_{po}^2 \frac{\partial^2 q}{\partial z^2} \end{aligned} \tag{4}$$

Or using the following matrix formulation (2D case):

$$\frac{\partial^2}{\partial t^2} \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} v_x^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial z^2} \\ v_n^2 \frac{\partial^2}{\partial x^2} & v_{po}^2 \frac{\partial^2}{\partial z^2} \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} \tag{5}$$

Wavefield snapshot in a homogeneous VTI medium



Impulse response computed using old equation with $v_s = 0$, along the symmetric axis and solved by REM. Homogeneous VTI medium with: $v_{po} = 3000m/s$, $\epsilon = 0.24$ and $\delta = 0.1$. p-wavefield (left) and q-wavefield (right).

Decoupled wave equations for P and SV waves - VTI media

- Recent works:
  - Liu et al. (2009) - P and SV–wave equations by factorizing coupled P-SV dispersion relation (Alkhalifah, 2000)
  - Etgen and Brandsberg-Dahl (2009) - Pure P wave equation (Harlan, 1995)
  - Du, Fletcher and Fowler (2010) - Pure P wave equation by factorizing coupled P-SV dispersion relation (Alkhalifah, 2000).

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Decoupled wave equations for P and SV waves - VTI media

- There are not new approximations for the Pure P-wave equation but these are all free from shear-wave artifacts
  - Harlan, 1995; Fowler, 2003; Etgen and Brandsberg-Dahl, 2009; Liu et al., 2009; Pestana et al., 2011
  - All these approximations are equivalent to what is usually known as Muir-Dellinger approximation (Dellinger and Muir, 1985; Dellinger et al., 1993; later reinvented by Stopin, 2001)
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Decoupled wave equations for P and SV waves - VTI media

Recently, Liu et al. (2009) factorized the dispersion relation presented by Alkhalifah (2000) and obtain two separate P- and SV-wave dispersion relations to

$$\omega^2 = \frac{1}{2}\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \pm \frac{1}{2}\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right] \left[1 + \frac{4v_{po}^2(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{\left[v_h^2\, k_r^2 + v_{po}^2 k_z^2\right]^2}\right]^{1/2} \qquad (6)$$

We expand the square root to first order ($\sqrt{1+X} = 1 + \frac{1}{2}X$) and obtain

Decoupled wave equations for P and SV waves - VTI media

P-Wave

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2\, k_r^2 + \frac{(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_h^2} \tag{7}$$

and

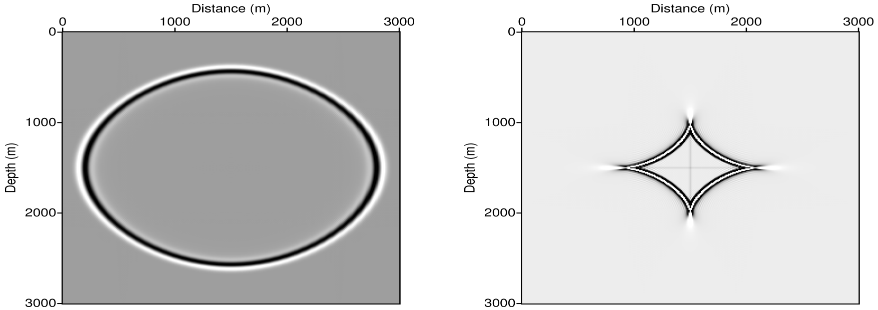
SV-wave

$$\omega^2 = -\frac{(v_n^2 - v_h^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{8}$$

where, here, $F = \frac{v_h^2}{v_{po}^2} = 1 + 2\epsilon$

For the equation for the SV-wave to be stable we must have that $v_h^2 - v_n^2 \geq 0$ or $\epsilon \geq \delta$.

Wavefield snapshot a homogeneous VTI medium



P-wave wavefield (left) and SV-wave wavefield (right) from decoupled P- and SV-wave equations proposed by Liu et at. (2009) also solved by REM.

Scalar wave equations in VTI media

We start with the exact dispersion relations for VTI media as derived by Tsvankin (1996):

$$\frac{v^2(\theta)}{v_{po}^2} = 1+\epsilon \sin^2\theta - \frac{f}{2} \pm \frac{f}{2}\left[1+\frac{2\epsilon\sin^2\theta}{f}\right]\left[1-\frac{2(\epsilon-\delta)\sin^2 2\theta}{f(1+\frac{2\epsilon\sin^2\theta}{f})^2}\right]^{1/2} \tag{9}$$

where θ is the phase angle measured from the symmetry axis. The plus sign corresponds to the P-wave and the minus sign corresponds to the SV-wave.
Here

$$f = 1 - \left(\frac{v_{so}}{v_{po}}\right)^2 \tag{10}$$

v_{po} and v_{so} are P- and S-wave velocities respectively, and ϵ and δ are the Thomsen (1986) parameters.

If one expands the square root to first order ($\sqrt{1-X} = 1 - \frac{1}{2}X$) and obtain the approximation

P-wave

$$\frac{v^2(\theta)}{v_{po}^2} = 1 + 2\epsilon \sin^2\theta - \frac{(\epsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\epsilon\sin^2\theta}{f})} \tag{11}$$

and

SV-wave

$$\frac{v^2(\theta)}{v_{po}^2} = 1 - f + \frac{(\epsilon-\delta)\sin^2 2\theta}{2(1+\frac{2\epsilon\sin^2\theta}{f})} \tag{12}$$

Equations (11) and (12), respectively, are equivalent to P8 and SV8 approximations present in the review paper of Fowler (2003).

With $\sin(\theta) = \frac{v(\theta)k_r}{\omega}$ and $\cos(\theta) = \frac{v(\theta)k_z}{\omega}$ and

$$v^2(\theta) = \frac{\omega^2}{k_r^2 + k_z^2} \tag{13}$$

The results are the dispersion relations

P-wave

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2\, k_r^2 - \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{14}$$

and

SV-wave

$$\omega^2 = v_{so}^2 (k_r^2 + k_z^2) + \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + F\, k_r^2} \tag{15}$$

where $v_h^2 = v_{po}^2(1 + 2\epsilon)$ and $v_n^2 = v_{po}^2(1 + 2\delta)$.

Here

$$F = 1 + \frac{2\epsilon}{f} = \frac{v_h^2 - v_{so}^2}{v_{po}^2 - v_{so}^2} \tag{16}$$

The new equations 14 and 15 are good approximations for the P- and SV-wave dispersion relation if

$$\left| \frac{2\,(\epsilon - \delta)\,\sin^2 2\theta}{f(1 + \frac{2\,\epsilon\,\sin^2\theta}{f})^2} \right| << 1 \tag{17}$$

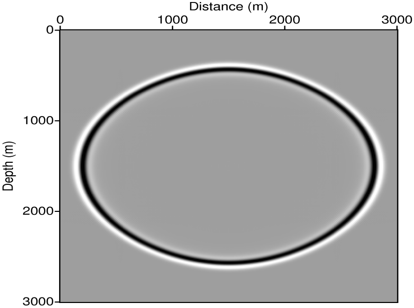
When we set $v_{so} = 0$ (or $f = 1$) equations 14 and 15 reduce to the equations 7 and 8, as derived from Alkhalifah (2000) by Liu et. al (2009).

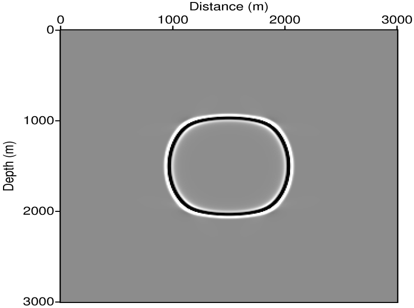
If we further set $\epsilon = 0$ in this expression, then $F = 1$, and equation 14 reduces to

$$\omega^2 = v_{po}^2 k_z^2 + v_h^2 k_r^2 - \frac{(v_h^2 - v_n^2)\, k_r^2\, k_z^2}{k_z^2 + k_r^2} \qquad (18)$$

which is the dispersion relation used by Etgen and Brandsberg-Dahl (2009) and Crawley et al. (2010) which can be credited to Harlan (1985).

Wavefield snapshot in a homogeneous VTI medium





P-wave wavefield (left) and SV-wave wavefield (right) from decoupled P- and SV-wave equations by the REM solving equations 14 and 15.

The P wave equation in the time-wavenumber domain for VTI media is given by:

$$\frac{\partial^2 P(k_r, k_z, t)}{\partial t^2} = -\left\{ v_{po}^2 k_z^2 + v_h^2 \, k_r^2 - \frac{(v_h^2 - v_n^2) \, k_r^2 \, k_z^2}{k_z^2 + F \, k_r^2} \right\} P(k_r, k_z, t) \tag{19}$$

The solution is:

$$p(t + \Delta t) + p(t - \Delta t) = 2 \cos(\Phi \Delta t) p(t) \tag{20}$$

where:

$$\Phi^2 = v_{po}^2 k_z^2 + v_h^2 \, k_r^2 - \frac{(v_h^2 - v_n^2) \, k_r^2 \, k_z^2}{k_z^2 + F \, k_r^2}$$

Since cosine is an even function, its expansion contains only powers of Φ^2

In order to have an efficient numerical scheme, we require that

$$\Phi^2 = \sum_j f_j(\vec{x})\, g_j(\vec{k})$$

so that

$$\Phi^2 p = \sum_j f_j(\vec{x})\, FFT^{-1}\{\, g_j(\vec{k})\, FFT(p)\}$$

where FFT and FFT^{-1} denote forward and inverse spatial Fourier transform.
Such separation is possible only if F is a constant, independent of $\vec{x}$.

The cosine function is given by (Kosloff et. al, 1989)

$$\cos(\Phi\Delta t) = \sum_{k=0}^{M} C_{2k}\, J_{2k}(R\,\Delta t)\, Q_{2k}\left(\frac{i\Phi}{R}\right) \tag{21}$$

Chebyshev polynomials recursion is given by:

$$Q_{k+2}(z) = (4z^2 + 2)\, Q_k(z) - Q_{k-2}(z)$$

with the initial values: $Q_0(z) = 1$ and $Q_2(z) = 1 + 2z^2$

For 3D case: $R = \pi v_{max} \sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}$,

The summation can be safely truncated with a $M > R\,\Delta t$ (Tal-Ezer, 1987).

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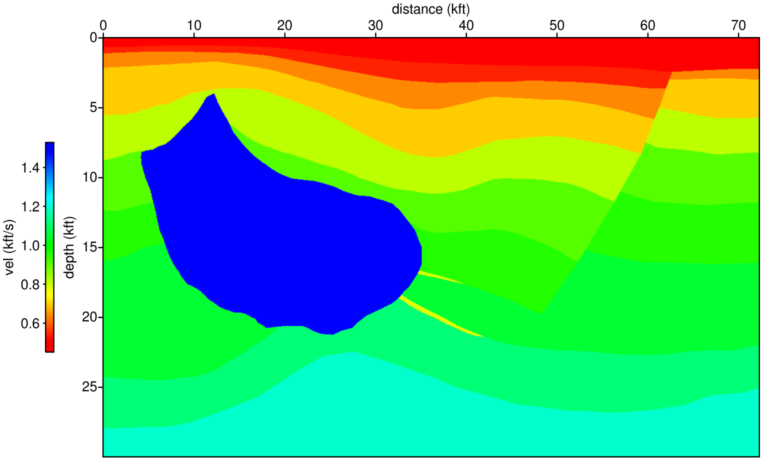
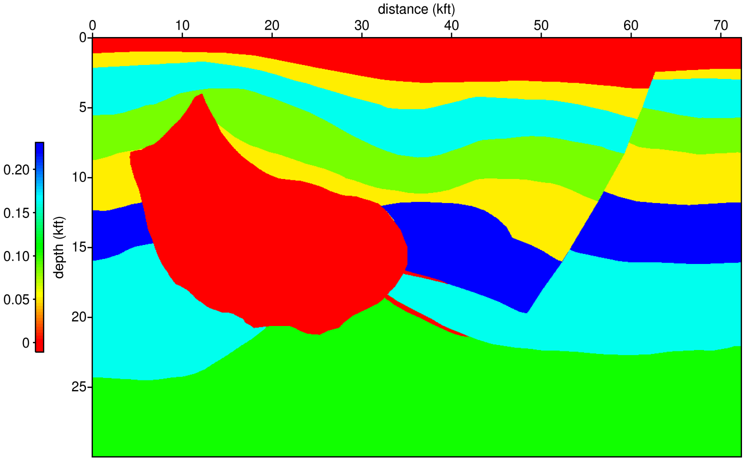
Velocity model



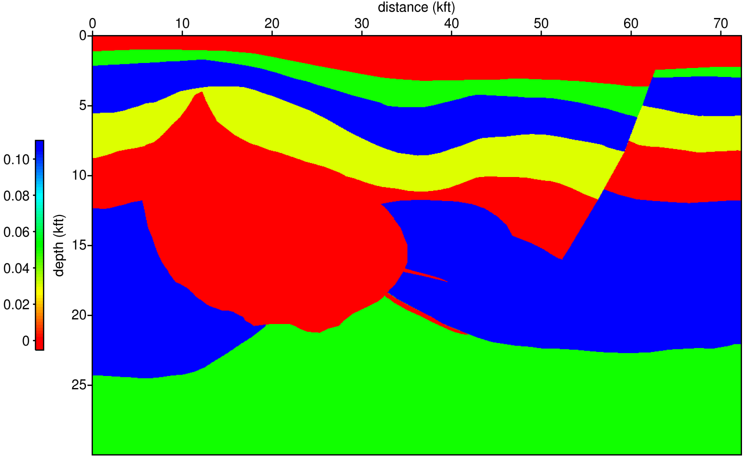
Hess VTI model

Epsilon parameter



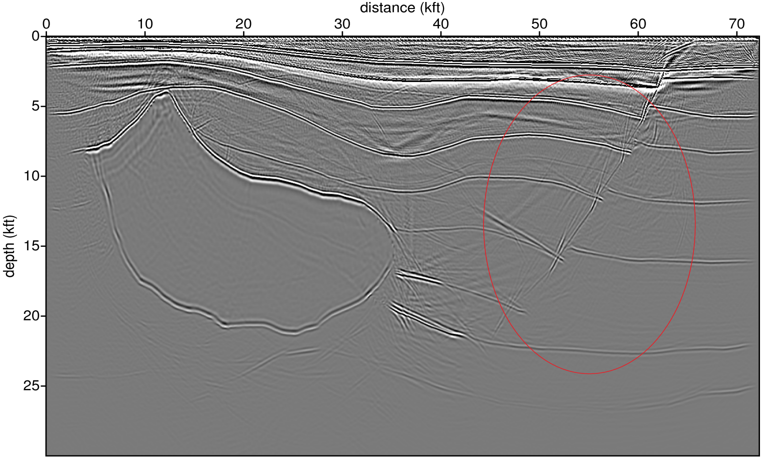
Hess VTI model

Delta parameter

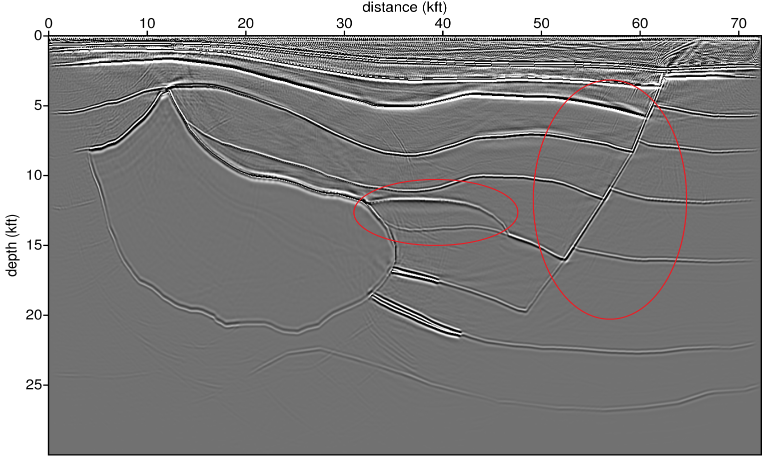


Hess VTI model

Isotropic RTM solved by REM (Δt=8 ms; F_{max}=35 Hz)



Anisotropic RTM solved by REM using the pure P-wave equation



Conclusions

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- We note no numerical noise from shear in the P wave acoustic VTI equation.
- Correct shear wave in the shear VTI wave equation
- The REM solution provides accurate and non-dispersive wave propagation. RTM using REM and pseudo-spectral methods for VTI media provides accurate images.

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