Time-steping evolution of wave-equation using a Laguerre polynomial expansion scheme.

Eduarda Rego[1], Reynam Pestana[1] and Edvaldo Araujo[1]

[1]Federal University of Bahia
CPGG/UFBA and INCT-GP/CNPq

85th SEG Meeting
18-23 October 2015

New Orleans, LA

Summary

Objective

- To propose an alternative solution to the acoustic wave equation using the orthogonal Laguerre polynomials;
  - Cosine operator in the two-step wave equation solution is expanded using Laguerre polynomials;
  - This new solution allows us to use a larger time-step than in the conventional methods (FD, PS);

Objective

Objective

- To propose an alternative solution to the acoustic wave equation using the orthogonal Laguerre polynomials;
  - Cosine operator in the two-step wave equation solution is expanded using Laguerre polynomials;
  - This new solution allows us to use a larger time-step than in the conventional methods (FD, PS);

Objective

Objective

- To propose an alternative solution to the acoustic wave equation using the orthogonal Laguerre polynomials;
  - Cosine operator in the two-step wave equation solution is expanded using Laguerre polynomials;
  - This new solution allows us to use a larger time-step than in the conventional methods (FD, PS);

Finite Difference Method

- Numerical solution for the wave equation based on the Taylor series expansion.
- It has been common to use FD approximations for both the time and spatial evolution of wavefields.

Introduction

Finite Difference Method

- Numerical solution for the wave equation based on the Taylor series expansion.
- It has been common to use FD approximations for both the time and spatial evolution of wavefields.

Finite Difference Method

- Numerical solution for the wave equation based on the Taylor series expansion.
- It has been common to use FD approximations for both the time and spatial evolution of wavefields.

Finite difference issues:

- A limit on the marching time step size;
- Numerical dispersion.

Finite difference issues:

- A limit on the marching time step size;
- Numerical dispersion.

Finite difference issues:

- A limit on the marching time step size;
- Numerical dispersion.

Two step extrapolation equation

- Cosine term expansion
  - Taylor series
  - Ortogonal polynomials

Two step extrapolation equation

- Cosine term expansion
  - Taylor series
  - Ortogonal polynomials

Two step extrapolation equation

- Cosine term expansion
  - Taylor series
  - Ortogonal polynomials

Two step extrapolation equation

- Cosine term expansion
  - Taylor series
  - Ortogonal polynomials

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Chebyshev Polynomials

- Rapid Expansion Method (REM)
  - It is the base of the recursive solution;
  - It is more accurate than the usual finite difference schemes;
  - It is a stable method;
  - It can march in time with larger steps.

Introduction

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1;1]$
- Laguerre Polynomials
- Hermite Polynomials

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1; 1]$
- Laguerre Polynomials
- Hermite Polynomials

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1; 1]$
- Laguerre Polynomials
- Hermite Polynomials

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1;1]$
- Laguerre Polynomials
- Hermite Polynomials

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1;1]$
- Laguerre Polynomials
- Hermite Polynomials

Introduction

Ortogonal polynomials

- Chebyshev Polynomials
  - Argument bounded $[-1; 1]$
- Laguerre Polynomials
- Hermite Polynomials

Theory

Acoustic wave equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} + L^2 u(x,t) = f(x,t) \qquad (1)$$

where $-L^2 = c^2(x)\nabla^2$.

Solution (VOP)

$$P(x,t+\Delta t) + P(x,t-\Delta t) = 2cos(L\Delta t)P(x,t) + S(x,t\pm\Delta t) \qquad (2)$$

Theory

Acoustic wave equation

$$\frac{\partial^2 u(x,t)}{\partial t^2} + L^2 u(x,t) = f(x,t) \qquad (1)$$

where $-L^2 = c^2(x)\nabla^2$.

Solution (VOP)

$$P(x,t+\Delta t) + P(x,t-\Delta t) = 2cos(L\Delta t)P(x,t) + S(x,t\pm\Delta t) \qquad (2)$$

Hermite expansion

$$cos(L\Delta t) = \frac{e^{-iL\Delta t} + e^{iL\Delta t}}{2} \quad (3)$$

- Generating function (Arfken, 1985)

$$e^{-s^2+2sx} = \sum_{k=0}^{\infty} \frac{s^k}{k!} H_k(x) \quad (4)$$

- Exponential operator:

$$e^{-iL\Delta t} = e^{-(\Delta t/2\lambda)^2} e^{-(-i\Delta t/2\lambda)^2+2\lambda L(-i\Delta t/2\lambda)} \quad (5)$$

Here the arbitrary parameter λ is introduced for convenience.

Hermite expansion

$$cos(L\Delta t) = \frac{e^{-iL\Delta t} + e^{iL\Delta t}}{2} \tag{3}$$

- Generating function (Arfken, 1985)

$$e^{-s^2+2sx} = \sum_{k=0}^{\infty} \frac{s^k}{k!} H_k(x) \tag{4}$$

- Exponential operator:

$$e^{-iL\Delta t} = e^{-(\Delta t/2\lambda)^2} e^{-(-i\Delta t/2\lambda)^2+2\lambda L(-i\Delta t/2\lambda)} \tag{5}$$

Here the arbitrary parameter λ is introduced for convenience.

- Hermite expansion

$$e^{-iL\Delta t} = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-i)^k}{k!} \left(\frac{\Delta t}{2\lambda}\right)^k H_k(\lambda L) \qquad (6)$$

About the convergence of this series:

$$k > (e\Delta t/2\lambda)$$

- Cosine expansion

$$cos(L\Delta t) = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k} H_{2k}(\lambda L) \qquad (7)$$

Hermite expansion

- Hermite expansion

$$e^{-iL\Delta t} = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-i)^k}{k!} \left(\frac{\Delta t}{2\lambda}\right)^k H_k(\lambda L) \qquad (6)$$

About the convergence of this series:

$$k > (e\Delta t/2\lambda)$$

- Cosine expansion

$$cos(L\Delta t) = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k} H_{2k}(\lambda L) \qquad (7)$$

- Relation between Laguerre and Hermite polynomials

$$H_{2k}(x) = (-1)^k \, 2^{2k} \, k! \, \mathcal{L}_k^{-1/2}(x^2) \tag{8}$$

- Cosine expansion

$$\cos(L\Delta t) = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k} \; H_{2k}(\lambda L)$$

$$= \sum_{k=0}^{\infty} C_k \, (\lambda\Delta t) \, \phi_k(\lambda^2 L^2) \tag{9}$$

- Relation between Laguerre and Hermite polynomials

$$H_{2k}(x) = (-1)^k \, 2^{2k} \, k! \, \mathcal{L}_k^{-1/2}(x^2) \tag{8}$$

- Cosine expansion

$$\cos(L\Delta t) = e^{-(\Delta t/2\lambda)^2} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k} \; H_{2k}(\lambda L)$$

$$= \sum_{k=0}^{\infty} C_k \, (\lambda \Delta t) \, \phi_k(\lambda^2 L^2) \tag{9}$$

Laguerre expansion

- Expansion coefficient:

$$C_k(\lambda\Delta t) = e^{-(\Delta t/2\lambda)^2} \frac{k!\, 2^{2k}}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k}. \quad (10)$$

- Recurrence relation:

$$C_k(\lambda\Delta t) = \left[\frac{2}{2k-1}\right] \left(\frac{\Delta t}{2\lambda}\right)^2 C_{k-1}(\lambda\Delta t) \quad (11)$$

Laguerre expansion

- Expansion coefficient:

$$C_k(\lambda\Delta t) = e^{-(\Delta t/2\lambda)^2} \frac{k!\ 2^{2k}}{2k!} \left(\frac{\Delta t}{2\lambda}\right)^{2k} . \tag{10}$$

- Recurrence relation:

$$C_k(\lambda\Delta t) = \left[\frac{2}{2k-1}\right] \left(\frac{\Delta t}{2\lambda}\right)^2 C_{k-1}(\lambda\Delta t) \tag{11}$$

- Expansion coefficient:

$$\phi_k(\lambda^2 L^2) = \mathcal{L}_k^{-1/2}(\lambda^2 L^2) \quad (12)$$

- Recurrence relation:

$$\phi_k(x^2) = \frac{(2k - 3/2 - x^2)}{k}\phi_{k-1}(x^2) - \frac{(k - 3/2)}{k}\phi_{k-2}(x^2) \quad (13)$$

$$\phi_0(x^2) = 1 \quad \text{and} \quad \phi_1(x^2) = (1/2 - x^2)$$

Laguerre expansion

- Expansion coefficient:

$$\phi_k(\lambda^2 L^2) = \mathcal{L}_k^{-1/2}(\lambda^2 L^2) \tag{12}$$

- Recurrence relation:

$$\phi_k(x^2) = \frac{(2k - 3/2 - x^2)}{k}\phi_{k-1}(x^2) - \frac{(k - 3/2)}{k}\phi_{k-2}(x^2) \tag{13}$$

$$\phi_0(x^2) = 1 \quad \text{and} \quad \phi_1(x^2) = (1/2 - x^2)$$

Laguerre expansion

- Expansion coefficient:

$$\phi_k(\lambda^2 L^2) = \mathcal{L}_k^{-1/2}(\lambda^2 L^2) \tag{12}$$

- Recurrence relation:

$$\phi_k(x^2) = \frac{(2k - 3/2 - x^2)}{k}\phi_{k-1}(x^2) - \frac{(k - 3/2)}{k}\phi_{k-2}(x^2) \tag{13}$$

$$\phi_0(x^2) = 1 \quad \text{and} \quad \phi_1(x^2) = (1/2 - x^2)$$

Wave equation solution

$$P(x,t+\Delta t)+P(x,t-\Delta t) = 2cos(L\Delta t)P(x,t)+S(x,t\pm\Delta t) \quad (14)$$

- Recursive solution

$$P(x,t+\Delta t)+P(x,t-\Delta t) = 2\sum_{k=0}^{\infty} C_k(\lambda\Delta t)\phi_k(\lambda^2 L^2)P(t) + S(x,t\pm\Delta t) \quad (15)$$

Wave equation solution

- Second order approximation in time

$$P(x,t+\Delta t) + P(x,t-\Delta t) = 2\alpha P(x,t) - \beta \Delta t^2 L^2 P(x,t) + S(x, t \pm \Delta t) \quad (16)$$

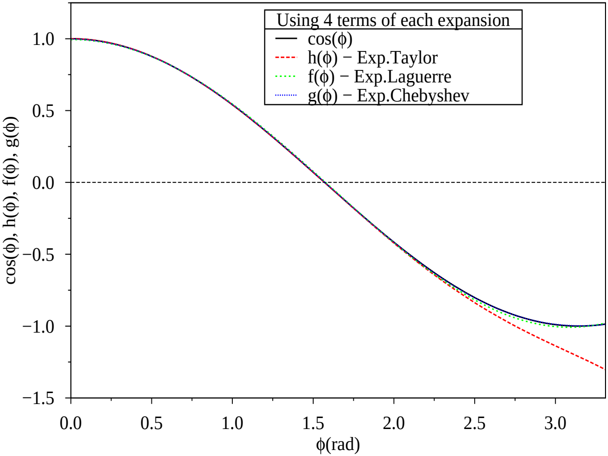
where

$$\alpha = C_0[1 + (\Delta t/2\lambda)^2] \quad \text{and} \quad \beta = C_0 = e^{-(\Delta t/2\lambda)^2}$$

.

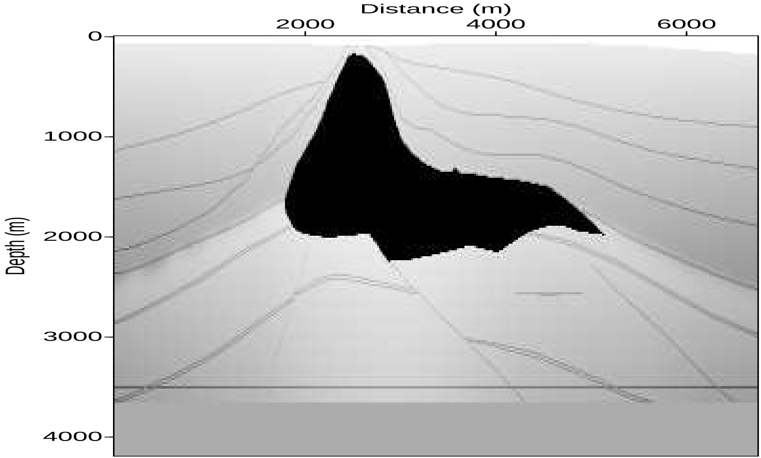
$$R = \pi c \sqrt{(1/\Delta x^2) + (1/\Delta z^2)} \quad \text{and} \quad \lambda = N/R$$

.



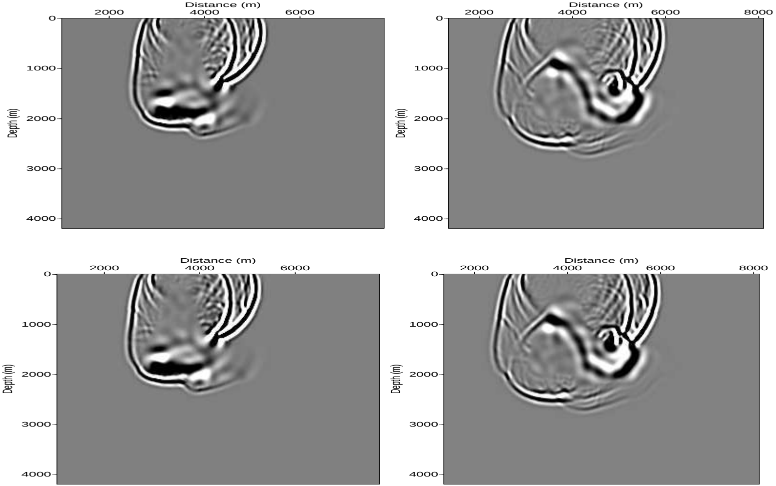
Plot with the results of the expansions of the cosine function using 4 terms by: Taylor series, Chebyshev and Laguerre polynomial expansions.

Seismic Modeling Results



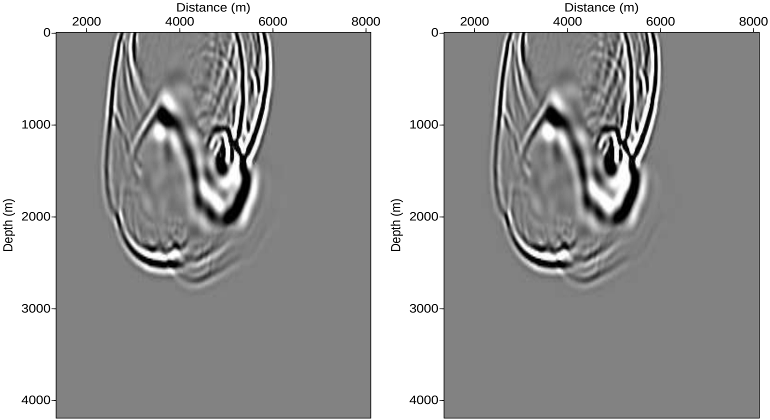
P-velocity model for the salt dome model.

Seismic Modeling Results



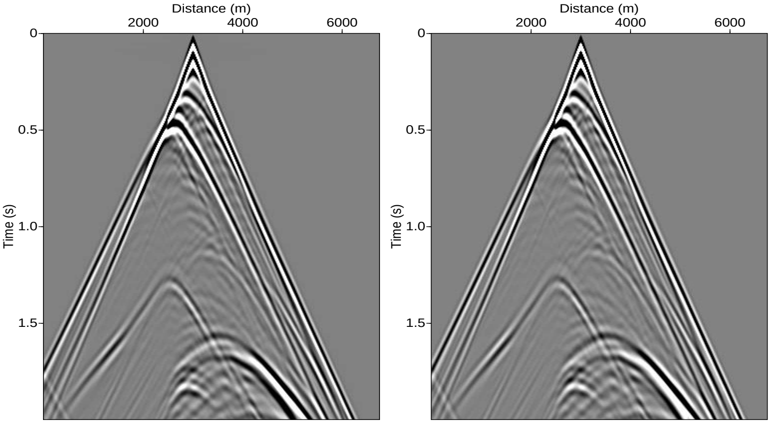
Snapshots at the times of: 0.8 s and 1 s using Chebyshev expansion (top) and Laguerre expansion (bottom). Maximum frequency of 25 Hz and with a time stepping of 2 ms.

Seismic Modeling Results



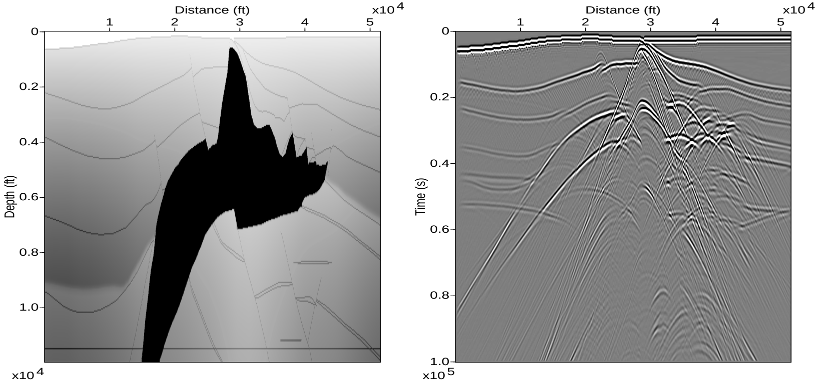
Snapshots generated from the salt dome model using Chebyshev (a) and Laguerre expansion (b) at the time of 1s. Maximum frequency of 25 Hz and with a time stepping of 2 ms .

Seismic Modeling Results



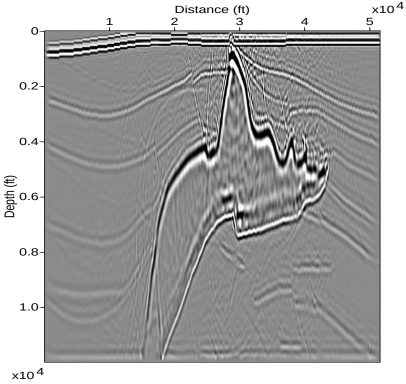
Seismogram generated from the salt dome model using the Chebyshev and Laguerre expansion with time stepping of 2 ms. The data was recorded at the depth of 20 m .

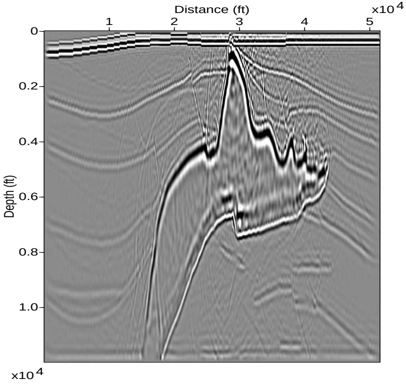
Seismic Migration Results



EAGE-SEG model and zero offset section.

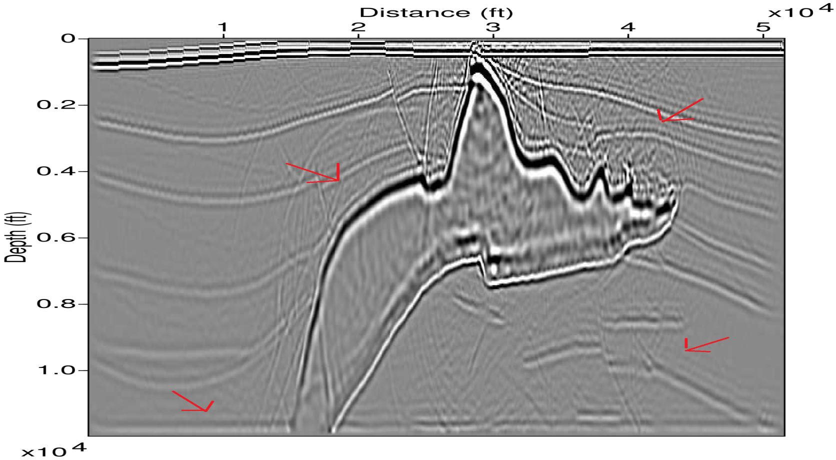
Seismic Migration Results





RTM results of the EAGE-SEG data set using 5 and 10 recursion terms.

Seismic Migration Results



RTM result of the EAGE-SEG data set using 5 recursion terms highlighting the imaged structures.

Conclusions

- Approximations for the cosine operator using Hermite and Laguerre polynomials;
- Satisfactory results for seismic modeling of complex models, stable and free of dispersion noise;
- RTM of pos-stack dataset with high quality results;
- Parameter λ was set equal to $8/R$ which ensured best results.

Acknowledgements

Acknowledgements

This research was supported by CNPq and INCT-GP/CNPq. The facility support from CPGG/UFBA is also acknowledged.