Fourier finite difference method for reverse time migration

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Outline

- Introduction
- Wave equation solution
    - Wave equation - an exact solution
    - Classical finite-difference (FD)
    - Laplace evaluation
- Pseudodifferential extrapolation operator
- Phase-shift plus interpolation and FFD methods for RTM
- Numerical results - Marmousi and Sigsbee2A datasets
- Conclusions
- Acknowledgments

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- Reverse time migration (RTM) is a depth migration algorithm. By using the full wave equation, RTM implicitly includes multiple arrival paths and has no dip limitation, enabling the imaging of complex structures.
- RTM produces images which are typically low frequency or require large computational resource (fine sampling).
- The fine sampling requirements occurs because finite-difference operators propagate high frequencies with an incorrect dispersion relation.

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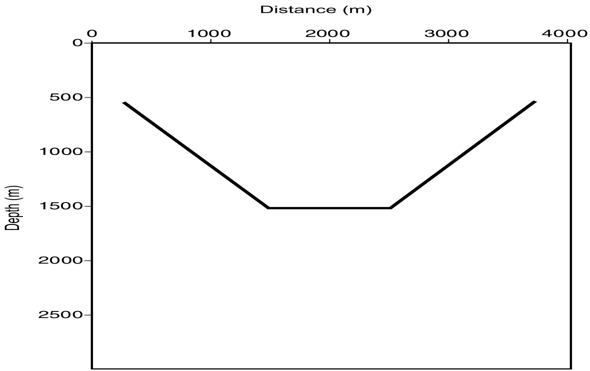
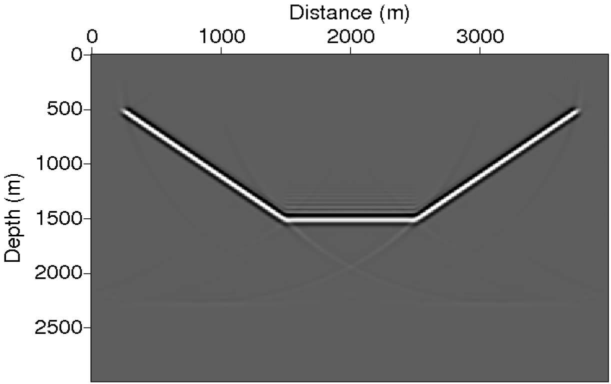
Velocity model



Simple constant velocity model

Finite difference method - 2nd order in time and 4th space
(Δt=1 ms, Δx=10 m, F_{max}=50 Hz)



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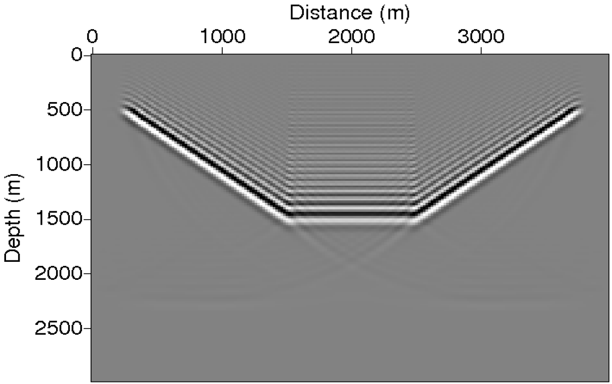
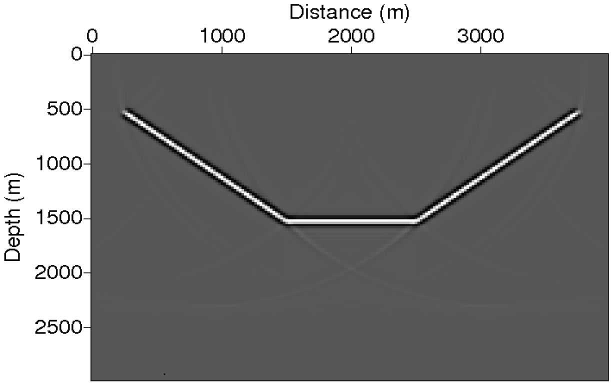


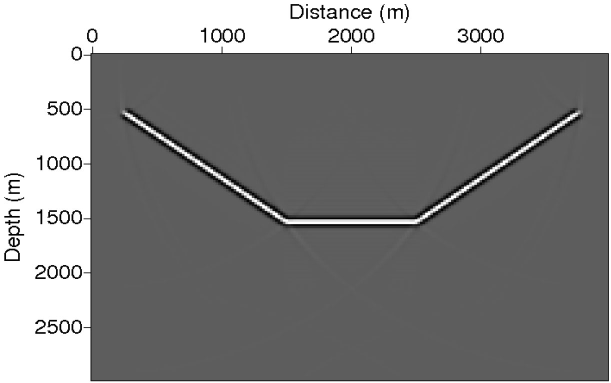
Simple constant velocity model

Fourier method - (Δt=2 ms, Δx=20 m, F_{max}=50 Hz)



Simple constant velocity model

Fourier method - (Δt=4 ms, Δx=20 m, F_{max}=50 Hz)



Acoustic wave equation

The acoustic wave equation in a source free medium with constant density is

$$\frac{\partial^2 p}{\partial t^2} = -L^2 p; \quad \text{with} \quad -L^2 = v^2 \nabla^2 \tag{1}$$

where $\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right)$ is the Laplacian operator,
$p = p(\mathbf{x}, t)$ is the pressure and $\mathbf{x} = (x, y, z)$ and
$v = v(\mathbf{x})$ is the compressional-wave velocity.

Equation (1) is a second order differential equation in the time variable.

Acoustic wave equation - An exact solution

Taking the wave equation (1)

$$\frac{\partial^2 p}{\partial t^2} = -L^2 p; \quad \text{with} \quad -L^2 = c^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \tag{2}$$

Initial conditions:

$$p(t=0) = p_0 \qquad \frac{\partial p}{\partial t}(t=0) = \dot{p}_0$$

Solution:

$$p(t) = \cos(L\,t)\,p_0 + \frac{sin(L\,t)}{L}\,\dot{p}_0 \tag{3}$$

The wavefields $p(t+\Delta t)$ and $p(t-\Delta t)$ can be evaluated by equation (3). Adding these two wavefields results in:

$$p(t+\Delta t) + p(t-\Delta t) = 2\cos(L\Delta t)\,p(t) \tag{4}$$

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Standard finite-difference schemes

$$p(t+\Delta t) + p(t-\Delta t) = 2\cos(L\Delta t)\, p(t)$$

Taking the Taylor series expansion of $\cos(L\Delta t)$.

Second order: $(1 - \frac{(L\Delta t)^2}{2})$

Fourth order: $(1 - \frac{(L\Delta t)^2}{2} + \frac{(L\Delta t)^4}{24})$

We obtain:

$$p(t+\Delta t) - 2\,p(t) + p(t-\Delta t) = -\Delta t^2 L^2 p(t) \qquad (5)$$

$$p(t+\Delta t) - 2\,p(t) + p(t-\Delta t) = -\Delta t^2 L^2 p(t) + \frac{\Delta t^4}{12} L^4 p(t) \quad (6)$$

Standard finite-difference schemes (Etgen,1986; Soubaras and Zhang, 2008).

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Laplace evaluation: $-L^2 = v^2\nabla^2$

- Fourier transformation scheme :

$$\frac{\partial^2 P}{\partial x^2} = IFT[-k_x^2\, FT[P(x)]]$$

- Finite difference:

$$\frac{\partial^2 P_j^n}{\partial x^2} \approx \frac{\delta^2 P_j^n}{\delta x^2} = \frac{1}{\Delta x^2} \sum_{l=-N}^{N} C_l\, P_{j+l}^n$$

- Convolutional filter (FIR):
  2nd order derivative on regular grids is replaced with a convolutional Finite Impulse Response filter

$$FIR(l) = D_2(l) * H(l)$$

where $D_2(l) * H(l)$ is a Hanning tapered version of the standard operator $D_2(l)$

Time stepping extrapolation

The wavefields $p(\mathbf{x}, t+\Delta t)$ and $p(\mathbf{x}, t-\Delta t)$ can be evaluated by equation (4), using the following form:

$$p(\mathbf{x}, t+\Delta t)+p(\mathbf{x}, t-\Delta t) = \\ = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} P(\mathbf{k}, t)\, \Omega(\mathbf{x}, \mathbf{k}, \Delta t)\, e^{i(\mathbf{k}\cdot\mathbf{x})}\, d\mathbf{k} \qquad (7)$$

where

$$\Omega(\mathbf{x}, \mathbf{k}, \Delta t) = 2\cos[L(\mathbf{x}, \mathbf{k})\, \Delta t]$$

and

$$L(\mathbf{x}, \mathbf{k}) = v(\mathbf{x}) \sqrt{k_x^2 + k_y^2 + k_z^2} \qquad (8)$$

is the pseudodifferential operator derived by Zhang and Zhang, 2009.

Separable Approximation

Normally, operators used in seismic imaging can be approximated as a series of separable terms such as

$$\Omega(\mathbf{x}, \mathbf{k}, \Delta t) \approx \sum_{j=0}^{n} a_j(\mathbf{x})\, b_j(\mathbf{k}) \qquad (9)$$

where n is the number of terms in the series. Thus, extrapolation in time is then approximated by

$$p(\mathbf{x}, t + \Delta t) + p(\mathbf{x}, t - \Delta t) \approx$$
$$\approx \sum_{j=0}^{n} a_j(\mathbf{x}) \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} P(\mathbf{k}, t)\, b_j(\mathbf{k})\, e^{i(\mathbf{k}\cdot\mathbf{x})}\, d\mathbf{k} \qquad (10)$$

Interpolation method

The time wave propagation can be performed in the following way:

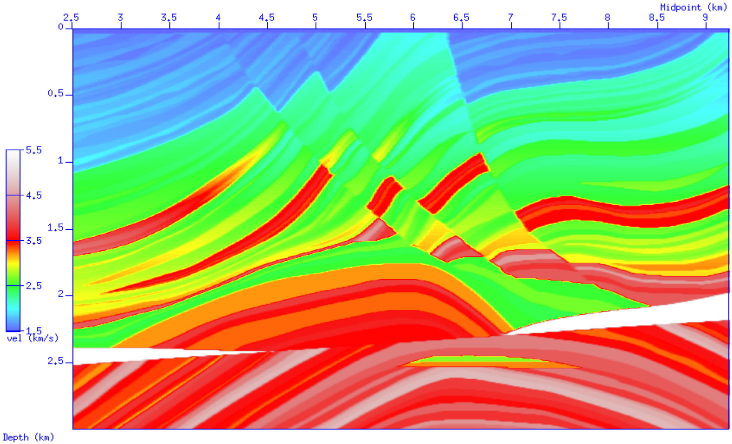
$$p(\mathbf{x}, t+\Delta t) = -\,p(\mathbf{x}, t-\Delta t) + \\ + \sum_{j=1}^{n} a_j(v)\, FFT^{-1}\, b_j(\mathbf{k})\, FFT\, p(\mathbf{x}, t) \qquad (11)$$

For the 2D case, each $b_n(k)$ is given by

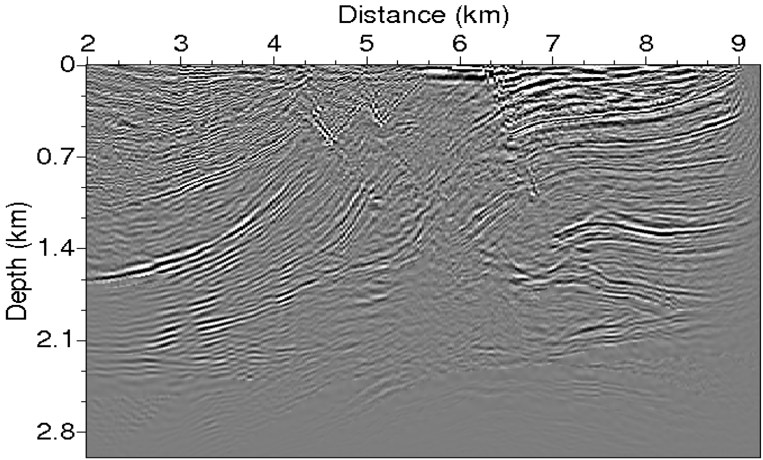
$$b_n(k) = \cos(v_n \sqrt{k_x^2 + k_z^2}\,\Delta t)$$

For each marching time step, this method requires one fast Fourier transform (FFT) and n inverse fast Fourier transforms (IFFT).

Velocity model

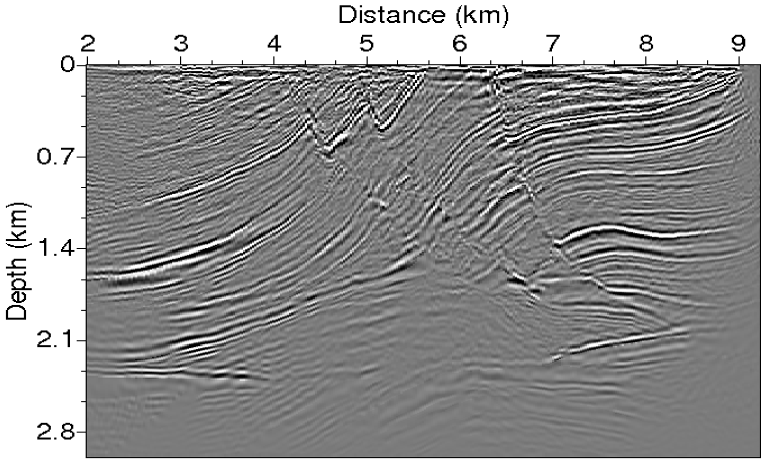


Reverse time migration - PSPI method - 3 velocities

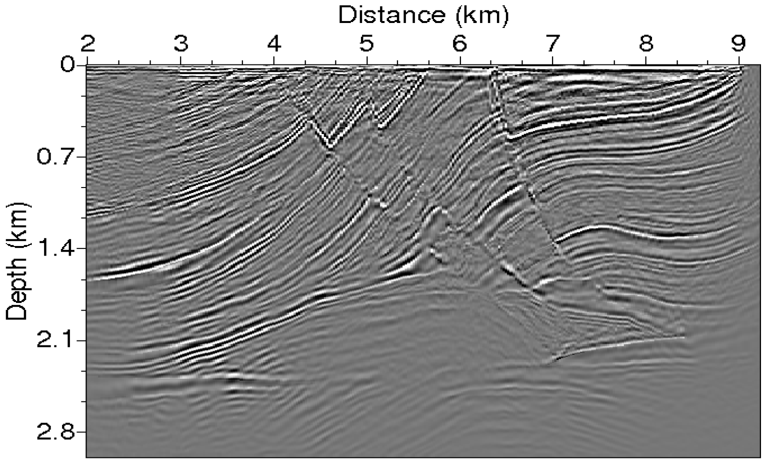


Marmousi Dataset

Reverse time migration - PSPI method - 5 velocities



Reverse time migration - PSPI method - 10 velocities



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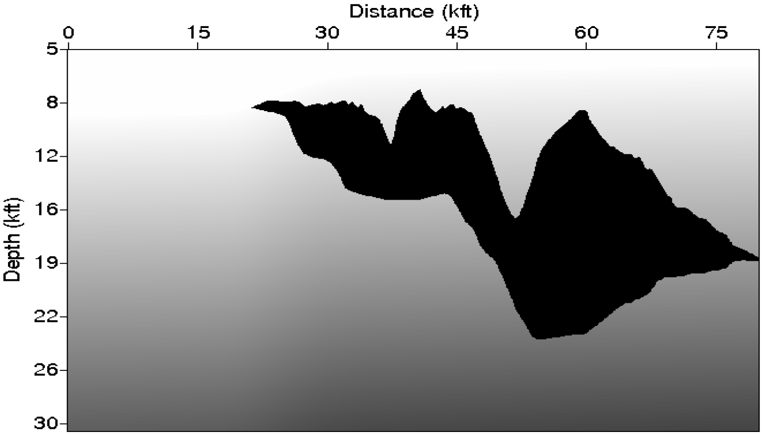

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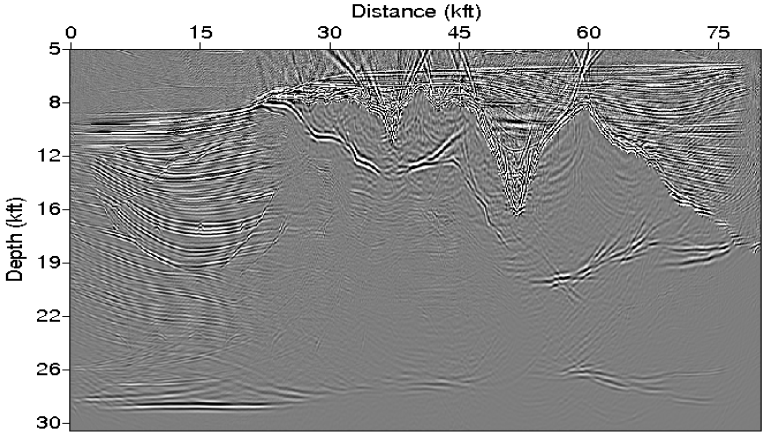

Sigsbee2A Dataset

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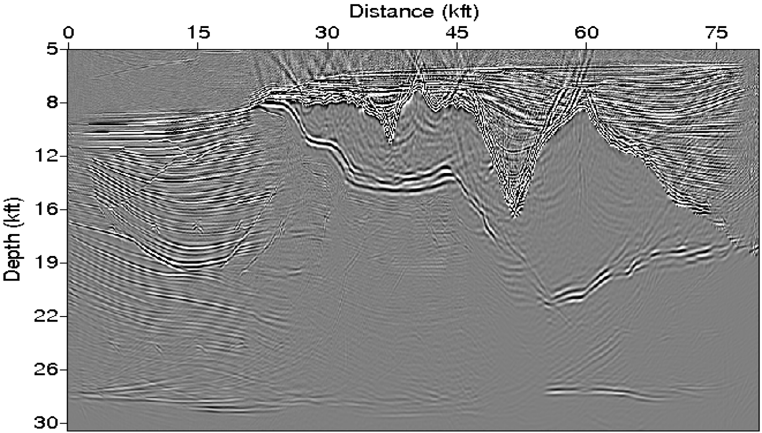

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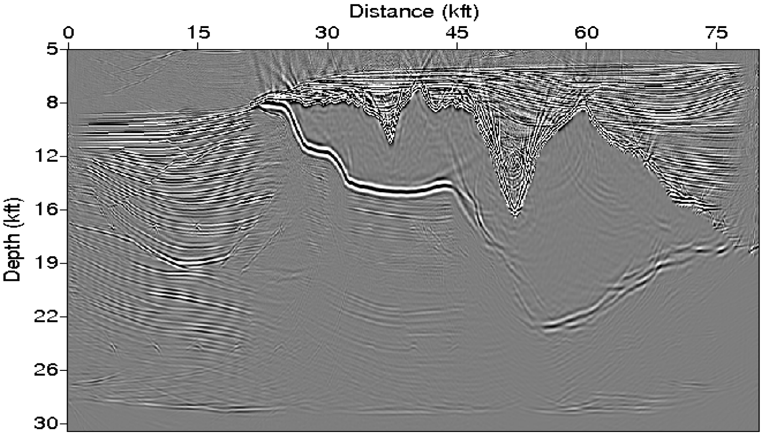
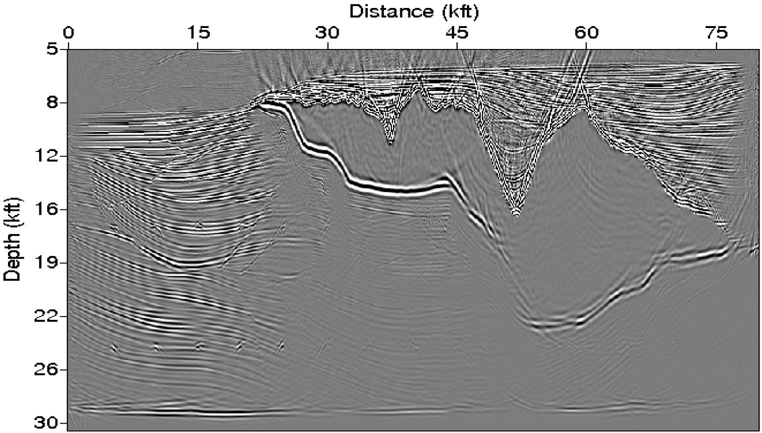
Reverse time migration - PSPI method - 8 velocities



Reverse time migration - PSPI method - 15 velocities



Fourier Finite Difference Method

Now we rewrite the equation (4) in the following form:

$$P(\mathbf{x}, t + \Delta t) + P(\mathbf{x}, t - \Delta t) = \cos(L\Delta t)\sec(L_0\Delta t)\,T(\mathbf{x}, t) \quad (12)$$

where $T(\mathbf{x}, t) = 2\cos(L_0\Delta t)\,P(\mathbf{x}, t)$, $L_0^2 = -\,v_0^2\,\nabla^2$ and v_0 is the minimum velocity of the media.

Using Taylor series, for both the $\cos(L\Delta t)$ and $\sec(L_0\Delta t)$ functions and substituting these approximations into equation (12) results in:

$$\begin{aligned} P(\mathbf{x}, t + \Delta t) + P(\mathbf{x}, t - \Delta t) &= \big[1 + c_2(\mathbf{x})K^2\Delta t^2 \\ + \quad c_4(\mathbf{x})K^4\Delta t^4 + c_6(\mathbf{x})K^6\Delta t^6 &+ \ldots\big]\,T(\mathbf{x}, t) \end{aligned} \quad (13)$$

where $K = \sqrt{-\nabla^2}$ or in the Fourier domain we have $K^2 = k_x^2 + k_z^2$

Fourier Finite Difference Method

Thus the equation (13) is rewritten as

$$\begin{aligned} P(\mathbf{x}, t+\Delta t) &= T(\mathbf{x}, t) - P(\mathbf{x}, t-\Delta t) + \left[c_2(\mathbf{x})\, K^2 \Delta t^2 \right. \\ &+ \left. c_4(\mathbf{x})\, K^4\, \Delta t^4 + \ldots \right]\, T(\mathbf{x}, t) \end{aligned} \quad (14)$$

with the c coefficients given by:

$$c_2(\mathbf{x}) = \frac{v_0^2}{2}\left\{1 - \alpha^2(\mathbf{x})\right\},$$
$$c_4(\mathbf{x}) = \frac{v_0^4}{24}\left\{5\; - 6\,\alpha^2(\mathbf{x}) + \alpha^4(\mathbf{x})\right\},$$
$$c_6(\mathbf{x}) = \frac{v_0^6}{720}\left\{61\; - 75\,\alpha^2(\mathbf{x}) + 15\alpha^4(\mathbf{x}) - \alpha^6(\mathbf{x})\right\},$$

and $\alpha(\mathbf{x}) = \frac{v(\mathbf{x})}{v_0}$.

Fourier finite-difference implementation

The equation (14) can be implemented in the Fourier domain and space domain in two steps as in the Fourier finite difference method

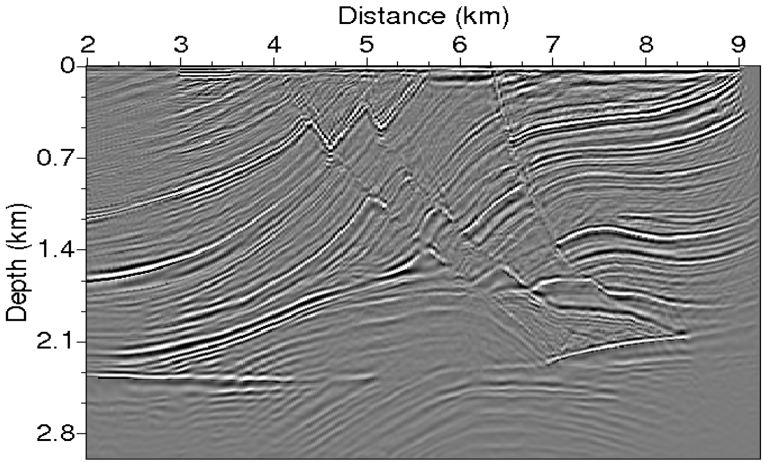
Considering only the first order velocity correction term on the RHS of equation (14) we have:

$$P(\mathbf{x}, t+\Delta t) = T(\mathbf{x}, t) - P(\mathbf{x}, t-\Delta t) - c_2(\mathbf{x})\Delta t^2 \nabla^2 T(\mathbf{x}, t) \quad (15)$$

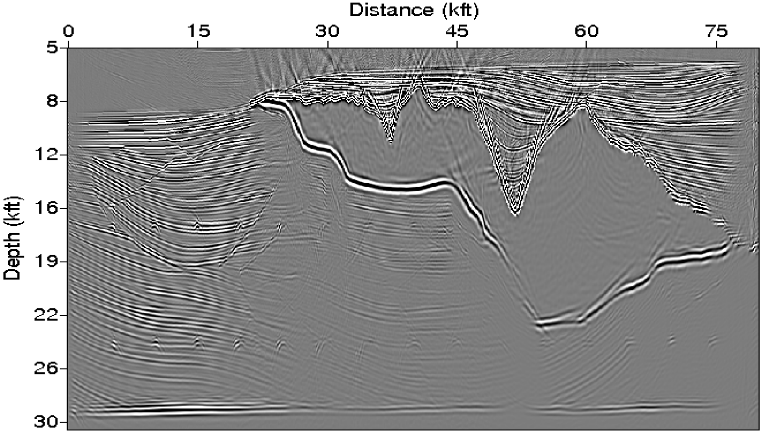
where ∇^2 is the Laplacian operator and it can be computed using

4th or higher order finite-difference schemes and $c_2(\mathbf{x})$ is the perturbation velocity computed for each spatial position as given before.

Reverse time migration - Fourier finite-difference method (FFD)



Reverse time migration - Fourir finite-difference method (FFD)



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- The first one is an interpolation procedure which is conceptually like a PSPI method for two way wave equation commonly used for the one way wave equation.
- We show that for complex structures we need 5 - 8 reference velocities to obtain reasonable seismic images.
- We also proposed a second method which is a FFD solution for the two wave equation.

Conclusions

- In this work we proposed two novel solutions for the two way wave equation for prestack RTM.
- The first one is an interpolation procedure which is conceptually like a PSPI method for two way wave equation commonly used for the one way wave equation.
- We show that for complex structures we need 5 - 8 reference velocities to obtain reasonable seismic images.
- We also proposed a second method which is a FFD solution for the two wave equation.
- The results obtained with Marmousi and Sigsbee2A datasets demonstrated the methods applicability and robustness.

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